\[ F = \frac{1}{\sum W \sin \alpha} \sum \left[ \frac{c' B + (W - uB) \tan \phi'}{\cos \alpha + (\sin \alpha \tan \phi' / F)} \right] \]

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \]
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Two-dimensional vorticity flow fields created in the wake of a plunging breaker were investigated for regular turbulent flow at a Reynolds number of 30,000. Velocity flow fields obtained from an earlier study that had employed digital particle image velocimetry were analysed to determine vorticity shedding patterns and the interactions between the vorticity filaments as flow progressed. Central difference approximations were applied to the velocity fields to determine vorticity at each point in the field. Most of the strong instantaneous vorticity observed in the flow field was in the form of filaments. A hierarchy of filaments of different lengths were observed, with the longest being as long as the height of the wave used. During the early phases of the flow, instantaneous vorticity tended to organise into thin filaments of counter-rotating pairs. Eventually, the co-rotating vorticity filaments coalesced and ultimately merged in the turbulent flow as flow progressed, while counter-rotating vorticity filaments were cancelled by viscous dissipation. The results suggested that filaments travel more slowly than the wave velocity and drifted towards the bed as they became elongated, and the number of filaments remaining in the flow were observed to decrease as flow progressed. Whereas phase-resolved instantaneous vorticity results showed pairs of counter-rotating vorticity filaments near the crest, the phase-averaged vorticity description of flow fields showed a dominant primary positive vorticity filament around the shear boundary layer.

INTRODUCTION

Wave breaking is one of the most important problems for coastal engineers. The breaking process leads to either surging, spilling or plunging breaker types. A plunging breaker is the most violent breaker, initiated when a steepening wave crest curls over to form an overturning jet. Steepening of the wave naturally induces high curvature and consequently strong vorticity. The jet plunges into the water ahead to form a plunger vortex, and also produces a splash-up leading to strong turbulence in the flow. The generated turbulence is a collection of weakly correlated vortical motions, which, despite their intermittent and chaotic distribution over a wide range of space and time scales, actually consist of local characteristic eddy patterns that persist as they move around under the influences of their own and other eddies’ vorticity fields (Hunt & Vassilicos 2000). Energy in the wave is transferred from the average flow to vortex structures at different scales. Due to the non-linear interactions between different scales, cascade processes of energy are very complex (Lin et al 2002). Wave-breaking turbulence accounts for most of the sediment suspension through the introduction of eddies associated with breaking waves, with increasing levels of turbulence coming towards the boundary layer from the water surface.

Ocean and coastal engineers have been interested in wave loading generated by turbulence and extreme waves and their interaction with marine structures. Ryu et al (2007) investigated potential damage to coastal structures caused by significant impacts of breaking waves and associated overtopping greenwater. The formation of bed topography, which, as a result of a complicated interaction between flow and sediment particles along the bed, has also received considerable attention. Lajeunesse et al (2010) used a high-speed video-imaging system to record the trajectories of the moving particles over a flat bed, and observed that entrained particles exhibit intermittent motion, composed of the succession of periods of flight and rest. Keshavarzi and Ball (2011) suggested that the structure of turbulent flow over the ripples in the bottom of an open channel is important for understanding of sediment particle entrainment and its transport. Two important issues which they suggested need to be understood in sediment movement are the stochastic nature of instantaneous shear stresses over a ripple bed, and how it influences on sediment entrainment and transport. They employed an Acoustic Doppler Velocity Meter (Micro-ADV) with a sampling rate of 50 Hz to measure three-dimensional velocities, a charge-coupled device camera to record images of particle motion, and image-processing techniques to provide an accurate measurement of flow structure and particle entrainment from the bed.
a study of sediment entrainment from the bed, Williams (1990) and Nelson et al (1995) investigated and found a high correlation between the stream-wise velocity component and the sediment flux. Additionally, Nelson et al (1995) found that the transport rate tends to be higher when the vertical velocity and Reynolds momentum flux are angled towards the bed. They found the best correlation between the sediment flux and the stream-wise velocity component to occur with a lag of 0.1 second, and consequently they suggested that a measuring frequency of 10 Hz would give the best results in terms of time scales. Nielsen (1984) showed experimentally that the process of sediment suspension is convective rather than diffusive, and that it is dominated by such features as travelling vortices, jets and turbulent bursts. In addition, the effect of wave-breaking on the concentration profile in terms of turbulence associated with each breaker type was discussed. Experimental results near the bed showed that sediment concentration profiles are very similar, suggesting that the near-bed reference concentration is independent of the external turbulence (breaking waves) and is controlled mainly by the bottom boundary layer. Voulgaris and Collins (2000) observed that an efficient numerical model of time-averaged suspended sediment concentration should be related to the breaking characteristics of the waves. A commonly used parameter to define the onset of wave-breaking is the ratio of wave height to water depth. Galvin (1972) defined another simple wave-breaking parameter that is a function of breaking wave height \( H_b \), beach slope \( \tan \beta \) and wave period \( T \):

\[
H_b = \frac{gT^2 \tan \beta}{B_b} \tag{1}
\]

For \( B_b > 0.068 \) the breaking waves are spilling, for \( 0.003 < B_b < 0.068 \) the waves are plunging, while surging breaking waves are present when \( B_b < 0.003 \). Galvin’s parameter has been used extensively in the study of the effect of the breaking-wave-induced vortices in sediment resuspension (Zhang & Sunamura 1990; Zhang 1994). Zhang (1994) identified four types (A–D) of oblique rolling vortices. The conditions for the occurrence of the observed types of vortices were defined as a function of Galvin’s breaker parameter (Equation (1)), and the breaking wave Reynolds number \( R_e = H_bL_b\nu \) where \( L_b \) and \( \nu \) are the wavelength at the breaking position and the seawater kinematic viscosity, respectively. From laboratory experiments, Zhang (1994) found that oblique vortices whirl the bed material and lift the sediment up into suspension, acting as a tornado. They observed that types A and B vortices last longer than types C and D. Type A horizontal vortices act in a similar manner as the oblique vortices, but are capable of re-suspending more sediment. Type B horizontal vortices dig the bed material like a cultivator, so that a larger amount of sand is lifted up into suspension and is entrapped in the vortex. Types C and D do not reach the bottom and thus do not contribute to sediment re-suspension. Considering the duration that the vortex is in contact with the bed, the strength of the vortex and thus its capability to re-suspend sediment was found to be highest in the B-type horizontal vortex, decreasing in the A-type horizontal vortex and the oblique vortex, in that order.

The creation of vorticity due to dissipating or breaking waves is a basic fluid dynamics phenomenon that is central to many geophysical and engineering applications (Buhler 2000). As stated by Long and Ozkan-Haller (2009), vorticity motions in the nearshore contribute to mixing and dissipation. The presence of vorticity induces shear at the water surface which changes the properties of the free surface waves when meeting an underlying current. Turbulent flows display intermittent pressure drops associated with vorticity concentrations, so pressure measurements can also be used to locate regions with high vorticity or dissipation in turbulent flows (Abry et al 1994). Brackett et al (1983) were the first to show, via simulations, that there is a direct relationship of the small pressure filaments with vorticity filaments. There have also been many experimental attempts to visualise coherent vorticity structures in boundary layers or in rotating flows, but only a few in the bulk of fairly homogeneous isotropic flows (Dernoncourt et al 1998). Sou and Yeh (2011) employed particle image velocimetry to investigate the fundamental characteristics of the flow structure in the vertical cross-shore plane as the wave evolves from the outer surf zone to the swash zone. They suggested that vorticity generated near the bed has a simple relation to the pressure gradient for boundary layer flows. The no-slip boundary condition, i.e. vanishing inertial force at the boundary, means that the phase-averaged momentum equation on a plane surface of a two-dimensional flow can be expressed (Sou & Yeh 2011) as:

\[
\mu \frac{\partial^2 \omega}{\partial z^2} = \frac{\partial \omega}{\partial x} = \frac{\partial P'}{\partial x} \tag{2}
\]

where \( \mu \) is the dynamic viscosity and \( P' \) is the excess pressure at the bed surface, given by \( P' = P + p g z \), where \( P \) is the atmospheric pressure, \( \rho \) is the water density, \( g \) is acceleration due to gravity and \( z \) is the water depth. Equation (2) implies that the flux of vorticity from the bed is created by the pressure gradient. This indicates that when the pressure gradient in the x-direction is negative, the vorticity gradient in the z-direction \( \partial \omega / \partial z \) is also negative, suggesting that the positive or clockwise (CW) vorticity is generated at the bed during uprush (Sou & Yeh, 2011). Figure 1 shows a schematic sketch by Sou & Yeh (2011) that summarises the observed flow structure and flow characteristics of vorticity observed in the surf and swash zones, which show the resulting load deposition, transport and suspension. They observed that in the surf zone a thin layer of counter-clockwise (CCW) vorticity on top of the CW vorticity is generated along the water surface, due to strong induction of the CW vorticity by the collision of the uprush and downwash flows. The surface flow pattern can support the presence of CCW vorticity on the surface behind the surf zone. They also pointed out that the presence of CCW vorticity on the surface is necessary, based on the Stokes theorem. Sou & Yeh (2011) also observed that flow separation occurs at the bore front, so the detached sediments from the bed by flow separation must remain suspended by the wave-breaking-generated turbulence and the shear-layer-generated turbulence. They also noted that the vorticity intensity at the shear layer decreases in magnitude in the offshore locations. In addition, suspended sediments should deposit on the bed with the flow attachment process as the
wave-breaking-generated turbulence and the shear-layer-generated turbulence levels become relatively low at the flow reversal.

Dabiri and Gharib (1997) studied the vorticity generation within a spilling water wave using the particle image velocimetry technique. Douady et al. (1991) studied turbulent Von Karman swirling flows using water seeded with air bubbles which displayed intermittent formation of filaments of bubbles, ascribed to vorticity concentrated on tube-like structures. Pressure measurements (Fauve et al. 1993; Abry et al. 1994) and pressure-velocity correlations (Cadot et al. 1995) were used in the detection of vorticity filaments, and in the study of their dynamic and statistical properties. Deroncourt et al. (1998) used an ultrasound scattering technique that allowed measurement of the vorticity in the bulk of the flow. They used short-time Fourier transforming of the incoming sound frequency and filtering of the scattered pressure signal, allowing the detection of large vorticity concentrations on small spatial scales, but not the study of their detailed internal dynamics. Their essential findings were that the vorticity at small scales has a clear tendency to exhibit intense rare events that are associated with localised structures compatible with the visualisation of low-pressure structures in the same flow. Lien and Sanford (2000) observed that turbulence vorticity flux plays an important dynamic role in the turbulence boundary layer. They used similarity scaling of the turbulence vorticity flux to estimate the bed stress, and observed that turbulence vorticity flux was related to the divergence of turbulence momentum flux, which represents the turbulence force on the mean flow. They argued that understanding the spectral properties of the turbulence vorticity flux would help improve turbulence parameterisation schemes in numerical models.

Kevlahan and Farge (1997) stated that, in order to successfully model a two-dimensional turbulent flow, it is essential to understand the role of vorticity filaments. They used the term ‘filament’ to refer exclusively to the thread-like structures observed in two-dimensional turbulence. Siggia (1981) was the first to show in a simulation that, in a turbulent flow, the regions in which vorticity had the largest modulus were in the shape of elongated filaments. Numerical studies by Jimenez et al. (1993) have also shown that intense vorticity levels in turbulent flows are organised in Burger-type filaments. The same objects have been identified by locating local minima of pressure fields in numerical experiments (Belin et al. 1996). Dritschel et al. (1991) noted that a common feature of strongly non-linear, high Reynolds number, two-dimensional flows is the presence of thin filaments of vorticity, while Jimenez and Wray (1998) also observed that most of the strong vorticity in the flow field is in the form of filaments. According to Elhmaidi et al (2004), vorticity filaments are easily generated during vortex–vortex interactions (such as vortex merger), and when vortices are exposed to strong shearing. Kimmoun and Branger (2007) used calculations of the spatial derivatives of the velocity field measured through particle image velocimetry, in order to obtain relevant information on vorticity. Dritschel (1989) observed that, once large vortices are formed in a turbulent flow, like-signed vortices merge together in an inelastic process. They observed that, during the merger process, filamentation occurs, leading to the formation of thin strips of vorticity. While some filaments remain attached to the large coherent vortices, others are expelled during the merger and remain detached from the large vortices. In a related study on vorticity filaments in two-dimensional turbulence Kevlahan and Farge (1997) suggested that vorticity filaments form in two-dimensional turbulence during the merger of coherent vortices, due to the strong shear forces that develop during the interaction. These forces pull some of the vorticity out into long, narrow strips of intense vorticity which are usually spiral in shape. Petitjeans (2003) also weighed in by pointing out that vorticity filaments are generated by roll-up of a fluid sheet around the vortex core, and that it is now well known that local stretching of vorticity in turbulent flows produces very intense vortices called vorticity filaments. Meunier et al. (2005) studied the physics of vortex merging. They observed that at the beginning of the merging, two strong filaments of vorticity are ejected, and roll up around the final vortex due to the differential rotation, leading to an axisymmetric vortex at late stages. Just as in previous work by Kevlahan and Farge (1997), Elhmaidi et al. (2004), Dritschel (1989), Jimenez et al. (1993), Jimenez and Wray (1998), Meunier et al. (2005) and Petitjeans (2003), narrow strips of intense vorticity are what is meant when referring to ‘vorticity filaments’ in this work. However, as pointed out by (Petitjeans 2003), the presence of vorticity does not imply the existence of a vortex, e.g. in shear boundary layers, where we may have vorticity but no vortices. Where a vortex is present, the term ‘vorticity’ describes its rate of rotation about some axis, locating and giving the vortex its strength or intensity.

Results presented herein are related to an earlier study in which experiments were performed in a laboratory wave flume to study the turbulence generated by 0.4 Hz breaking water waves (with a wave height of 12 cm) as they propagate along a glass-walled laboratory flume and break on a 1:20 plane slope. Characterisation of the flow structure, description of the experimental setup and methods used have been previously reported in Mukaro and Govender (2011; 2013). Here it will suffice to just mention that the temporal and spatial evolution of instantaneous and phase-ensemble-averaged velocity flow fields induced were examined using flow visualisation and digital correlation image velocimetry. A computer-driven electronic measurement system was designed, developed and employed to capture images of breaking waves. An eight-bit, monochromatic, progressive-scan digital camera was connected to the computer and mounted on the side of the flume to capture images of these breaking waves. Through the use of a trigger pulse from the wave generator, the computer synchronises image acquisition by the camera at the instance the computer drives the strobe lights to illuminate the field of view. The wave cycle was subdivided into 20 overlapping phases or fields of view. For a particular phase, 100 sequential image pairs were captured and the information saved on a computer. Instantaneous velocities were later determined by measuring the combined displacement of polystyrene particles and air bubbles entrained in the flow. A digital correlation image velocimetry technique was employed to calculate the spatial cross-correlation of the grey-scale image data by means of computing the cross-power spectrum of the Fourier-transformed image samples. This yielded 100 instantaneous velocity flow fields for each phase. Velocity measurements obtained from this turbulent flow enabled the computation of instantaneous and mean flow vorticity. A single component of vorticity is obtained from velocity fields by measuring the rotation of particles seeded in a turbulent flow. The velocity data were differentiated to yield vorticity field data that characterises the wake of a plunging breaker. The spatial and temporal evolution of both instantaneous vorticity and vorticity of the mean flow are presented only for phases where turbulence is predominant.

**VORTICITY CALCULATION**

Vorticity of a flow field is defined as the curl of the velocity field. It is a vector field that gives a microscopic measure of the rotation at any point in the fluid, and plays a vital role in the dynamics of turbulent fluid flows. The vorticity vector plays an outstanding role in turbulence kinematics and dynamics of the flow (Wang 2012), and has been described as the principal quantity that defines the flow structure (Kim et al. 1995; Wu et al. 2006). If
resolved temporally, the vorticity field can be much more useful in the study of flow phenomena than the velocity field (Raffel et al. 1998). At any instant eddies are present in the flow, ranging in size from the largest geometric scales of the flow down to small scales where molecular diffusion dominates. These eddies are continuously evolving in time, and the superposition of their induced motions leads to the fluctuating time records normally observed. By calculating the vorticity fields from the velocity field it is possible to follow the motion of coherent structures in the flow.

For a three-dimensional fluid flow, vorticity can be calculated in Cartesian coordinates, from the partial derivatives of the velocity components as:

\[
\vec{\omega} = \nabla \times \vec{u} = \hat{x} \left( \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \right) + \hat{y} \left( \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right) + \hat{z} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)
\]

where \( \hat{x}, \hat{y} \) and \( \hat{z} \) are the unit basis vectors for the three-dimensional Euclidean space and \( u, v \) and \( w \) are velocities in the \( x, y \) and \( z \) directions, respectively. The turbulent flow studied here has velocity fields confined only in the two-dimensional \( x-z \) plane. This means that only one component of vorticity is present. This component, \( \omega_y \), which is perpendicular to the flow and points in the orthogonal \( y \)-direction is given (Sou & Yeh 2011) as:

\[
\omega_y = \hat{y} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right).
\]

Figure 2 shows data points on a computational grid of a velocity mesh used for calculating vorticity at a particular point \((i, j)\). The four yellow-shaded grid points were used to numerically estimate vorticity at this point.

Instantaneous vorticity at a given grid point \((i, j)\) was computed from derivatives of the instantaneous horizontal, \( u_t \) and vertical, \( w_t \), velocity components using second-order central difference approximation (Sou & Yeh 2011; Lee & Lee 2001) as:

\[
\omega_y(i, j) = \frac{u_t(i, j + 1) - u_t(i, j - 1)}{2\Delta z} - \frac{w_t(i + 1, j) - w_t(i - 1, j)}{2\Delta x}.
\]

RESULTS AND DISCUSSIONS

Spatially and temporally resolved velocity vector fields previously measured for a plunging breaker were used to investigate the phase evolution of the instantaneous and mean flow vorticity. For easier visualisation, the vorticity components are presented in the form of contour plots, and show evolution of the vorticity as flow progressed. In order to describe the two-dimensional configuration of the wake of this turbulent flow, instantaneous vorticity and vorticity of the mean flow were computed from the instantaneous velocity fields. It should, however, be emphasised that most flows of practical interest are three-dimensional, so vorticity behaves very differently in three dimensions, compared to two.

Wave images

Figure 3 shows a series of typical grey-scale images of a 0.4 Hz plunging breaking wave captured by a monochrome digital camera at six different phases at one station, where turbulence was observed to be predominant. These photographs were captured at one position and were intended to illustrate the qualitative features of the breaking waves that were analysed and presented in this paper. The temporal evolution of the breaking is shown in two space coordinates in a series of photographs where the time separation between two consecutive frames is 0.125 s. The magnitude of the intensity of each pixel in the images is represented by levels of grey from black to white. Images from the 20 wave phases are labelled relative to the wave period, with \( t/T = 0.00 \) representing the first phase; 0.05 for the second, through to phase \( t/T = 0.95 \) for the twentieth phase. These correspond to phases: \( t/T = 0.00, 0.05, 0.10, 0.15, 0.20 \) and 0.25. The waves propagate from left to right, and the images evolve from left to right, top to bottom, as shown in Figure 3. The photo for phase 0.00 (top left) shows an image of the breaking wave at the phase when the crest is just coming into the field of view of the camera. The last photo in Figure 3 shows the image at a later phase when the crest has just passed. White polystyrene beads that were used as tracer particles together with air bubbles to determine the velocity flow fields are clearly visible in the images. The white cap in the images shows aerated water.

The breaking process is observed to create a high-speed roller that rides on the front face of the wave crest. The water mass in the roller is rotating about the horizontal axis that is parallel to the front face of the wave, creating a huge amount of turbulence and shear stress at the front face of the wave. Images at other phases show the trough part of the wave which is not as dynamic, so vorticity results only for the six phases presented.

Instantaneous vorticity \((\omega_y)\)

Characteristics of vorticity structures in the turbulent flow, and their evolution as phase progresses, are presented in Figure 4, which shows the sequence of contour plots of instantaneous vorticity generated from the images. These reveal the evolution of the vorticity fields as flow progressed. These
plots were obtained from the instantaneous velocity fields of the images presented in Figure 3. These six snapshots of the vorticity field are presented here to best illustrate spatio-temporal evolution of near-surface eddies during the passage of the wave crest. The colour bar shows the magnitude and direction of the vorticity. Colours are coded according to the intensity of the vorticity field from minimum vorticity (green) to the maxima (either red or blue). Positive vorticity indicates motion in clockwise rotation, and the direction is into the plane of the figure, while negative vorticity indicates anticlockwise rotation, with direction out of the plane of the figure. Positive vorticity has been conveniently taken to point in the positive y-direction. Red-filled contours indicate clockwise or positive vorticity, whereas the blue-filled contours represent counter-clockwise or negative vorticity. The abscissa shows distance from the intersection of the still water line with the beach slope, and is taken as negative away from the shore. The still water line is at elevation $z = 0$ cm on the contour plots.

The spatial variation of the instantaneous vorticity shows a series of strong counter-rotating filaments observed behind the leading edge of the breaker. The presence of vorticity patches near the water surface shows that wave-breaking which occurs at the free surface is the major source of vorticity in the flow. The underlying physical mechanism for this vorticity near the surface was explained by Sou and Yeh (2011), who suggested that water on the front face must move faster than the propagation speed, hence the formation of a surface roller. On the other hand, the water behind the front face must move more slowly than the propagation speed, creating a divergence on the surface. The filaments are observed to diffus into the bottom of the flume, reaching the bed after the crest has passed (phases 0.20 and 0.25). As pointed out by Longo (2008), deepening is quite fast immediately after breaking and is slower at subsequent phases. Pairs of counter-rotating filaments have peak vorticity of magnitude 100 s$^{-1}$ which is observed to decrease as flow progresses. These patches have only been shown to exist, but the mechanism by which they are set up is not clear. However, they are understood to dissipate the remaining wave energy. It is the interactions between these adjacent counter-rotating filaments that produce a wake of complex vortex distribution behind the wave crest. These filaments, firstly observed at elevations above $z = -5$ cm at phase 0.00, originate from the free surface and obliquely propagate in the direction of the wave. Their presence is an indication of strong mixing taking place there. Near the trough level there are occasional regions devoid of strong vorticity. Early phases also show large expanses of the fluid with nearly zero vorticity. Bakewell and Lumley (1967) and Aubry et al (1988) used the proper orthogonal decomposition in the near-wall region, and observed that a pair of counter-rotating stream-wise vortices contain the largest amount of energy. Ting (2006; 2008) used particle image velocimetry (PIV) to study instantaneous turbulent velocity fields associated with a broken solitary wave on a plane slope, and also observed that large eddies were composed of two counter-rotating vortices.

Just as parallel electric currents in the same direction attract one another because of their magnetic interaction, co-rotating filaments coalesce and ultimately merge in viscous flow as flow progresses, while counter-rotating vorticity filaments interfere with one another by viscous vorticity cancellation. Vorticity cancellation ultimately leaves predominantly weaker positive vorticity throughout most of the wake and negative vorticity confined to just below the saddle points, as seen in the last two panels of Figure 4. This cancellation is the key mechanism for the decay of two-dimensional vorticity distributions. Leweke et al (2016) illustrated this mechanism to show how, in a viscous flow, two identical co-rotating vortices ultimately merge, while counter-rotating ones cancel. Coalescing of co-rotating vorticity filaments is evidenced by the eventual formation of a positive vorticity filament ring observed to be centred around $x = -240$ cm in the last panel of Figure 4. Negative vorticity (anticlockwise) filaments are observed to disappear as a result of mixing, leaving the fluid volume filled mainly with positive, clockwise filaments. The filaments deepen and diffuse towards the bottom, as shown in phases 0.10–0.25. Saddle points near the crest are characterised by strong filaments of obliquely descending positive and negative vorticity below them.

Characteristics of instantaneous vorticity structures in the flow, such as the vorticity filament width (core diameter in three-dimensional vorticity), length and translational speed of the filaments between
frames as the structures travel downstream in the flow, have been estimated. It can be observed that instantaneous vorticity filaments are created in a large range of sizes. They are estimated to have individual length scales that stretch up to 12 cm (height of wave used) and a typical mean width of about 2.0 cm. Simulations by Jimenez and Wray (1998) have shown that the filaments can simultaneously have lengths of the order of the injection scale and a core diameter as small as the Kolmogorov scale. A pair of counter-rotating filaments has an estimated centre separation of about 2.0 cm. Vorticity analysis not only identifies any shearing motion, but also vortex cores present in the flow (Adrian et al. 2000). Kevlahan and Farge (1997) noted that most of the time filaments formed during a vortex merger tend to be distributed as a sequence of approximately circular rings around the coherent vortex. In the contour plot of phase 0.20, a large positive vorticity structure of about 60 s⁻¹ vorticity magnitude is observed, which is centred around \((x, z) = (–250, –7)\) cm. Results show this structure to have developed into a vortex ring that has evolved and moved to position \((x, z) = (–240, –7)\) cm in the plot of phase 0.25. Figure 5 shows the instantaneous velocity field that gave rise to vorticity contours presented in the last panel of Figure 4. The vortex ring is clearly visible in the velocity field.

The wave was divided into 20 phases and the period of wave used is 2.5 s. The time interval between two consecutive phases is 0.125 s. Thus the ring structure has moved about 10 cm in 0.125 s. This implies that the ring propagates at about 80 cm/s towards the shore, which is about 0.74 \(c\), where \(c = 108\) cm/s is the wave phase velocity. This is in agreement with Kimmoun and Branger (2007), who observed that vorticity filaments propagated obliquely towards the bottom, moving more slowly than the wave crest. Miller (1976) traced vortices and noted that they travelled more slowly than the wave velocity, and drifted downwards while expanding.

Figure 6 shows contours of instantaneous vorticity under the wave crests, measured at five different cross-shore positions along the flume. The Figure 6(e) measurements were captured at a point closest to the shore, with its centre only about 1.0 m from the shore, while the panel in Figure 6(a) is in the deepest water and furthest from the shore, just shoreward of the break point. The flume bed is located at different positions in the panels. While the size of vorticity filaments is observed to decrease towards the shore, there is a general increase in the intensity. Stretching a filament along its axis will make it rotate faster and decrease its width (or diameter in three-dimensions) in order to maintain its angular momentum constant. A well-known example in fluid mechanics is the bath tub vortex that becomes smaller and rotates faster as it goes from the fluid surface to the exit (Petitjeans 2003). Both negative and positive vorticity filaments of magnitude

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**Figure 5** Corresponding instantaneous velocity field for the vorticity field for phase \(t/T = 0.25\) shown in Figure 4

**Figure 6** Contour plots of instantaneous vorticity under the wave crests, measured at five different positions along the flume – (a) is furthest from the shore, while (e) is closest; note that there is no time relationship between these plots.
up to 100 s\(^{-1}\) can be observed impinging the flume bed as flow progresses from deep water to shallow water. Such high-vorticity filaments are responsible for lifting sediments from the bed and transporting them.

**Mean flow vorticity \(\Omega_0\)**

Figure 7 shows contour plots of the mean flow vorticity for the six phases of interest. The plots show that the mean flow is characterised by a single, strong, positive vorticity filament located below the crest, at the shear boundary layer. The shear layer arises from the fast-moving mixture of crest water/ bubbles and the opposing slow-moving water near the trough. Mean flow vorticity was obtained by averaging over 100 instantaneous vorticity fields at a particular phase to give the ensemble-averaged vorticity. Mean vorticity distribution shows moderate spatial concentration gradients, in contrast to the patchy instantaneous vorticity. During the early phases of the flow, there is a concentrated region of positive vorticity, about 5 cm thick near the breaker shear layer. This positive vorticity was due to the shear generated by the high-speed uprush flow overlapping a slower receding downwash layer of fluid (Misra et al 2005). Co-rotating instantaneous filaments observed earlier coalesce to become a larger and stronger single filament. Kevlahan and Farge (1997) observed that strong shearing associated with the vortex mergers produces intense filaments of vorticity. Thus mean flow organises with the formation of a large coherent positive vorticity filament near the shear boundary layer.

Even though the instantaneous flow between the shear layer and the free surface is very turbulent (Figure 4), the observed mean vorticity field is rather weak, except within the shear layer. This is consistent with observations by Lin and Rockwell (1994). Averaging instantaneous vorticity makes small-scale vorticity filaments vanish, so contours of the mean flow vorticity only show the development of large-scale structures within and around the shear layer, which develops and moves downstream. The fluid beneath the elevation of \(z = -5\) cm is relatively vorticity-free for the first two phases, but as flow progresses, it is observed that positive vorticity diffuses towards the flume bed. As observed by Sou and Yeh (2011) and presented in Figure 7, the fluid immediately behind the crest (for phases 0.10 to 0.25) is characterised by negative vorticity that rides above positive vorticity at elevation centred around \(z = -5\) cm. As pointed out by Dabiri and Gharib (1997), the negative vorticity seen on the wave, below the surface, and above the positive shear layer above \(z = 0\) cm, indicates the existence of a stagnation point at that location. In agreement with observations by Sou and Yeh (2011), the maximum intensity of mean vorticity occurs around the shear layer, and the intensity decreases in magnitude as flow progresses. Thus maximum positive vorticity will remain in the shear layer region between the upper, faster-moving part of the breaking wave and the quiescent region below. Below the trough level mean flow vorticity is about the same order of magnitude as the wave phase speed divided by the local water depth, \(c/h = \sqrt{gh}/h\).

For the same region, Chang and Liu, (1998; 1999) also found that the vorticity generated by wave breaking was of the same order of magnitude, as the phase speed divided by the local water depth.

Mean vorticity contour plots also reveal the phenomenon of vorticity-shedding at later phases of the flow, in which the tail of the initially strong boundary layer vorticity peels off, weakens and diffuses from the shear boundary layer, reaching the flume bed after the crest has passed. This results in the decay of the vorticity from the neighbourhood of the initially stronger region (Jimenez 2004). The original layer breaks in this way into nearly circular cores. The strong negative vorticity patches near the free surface directly above regions \(-255 \text{ cm} < x < -242 \text{ cm}\) for phase 0.10, and \(-242 \text{ cm} < x < -232 \text{ cm}\) for phase 0.15, may be responsible for the shedding observed. The explanation for the observed decay is that, at later phases, the effects of small-scale mixing due to the strong turbulence in the flow have greatly reduced the presence of patches of vorticity in the flow. It was shown by other researchers (e.g. Stanly et al 2002) that, while the large scales in the flow field adjust slowly to variations in the local mean velocity gradients, the small scales adjust rapidly.

**CONCLUSIONS**

Results have been presented of instantaneous and mean vorticity, aimed at determining their evolution as flow progressed. These were estimated from the measured velocity fields using a central difference method. Results have shown that the spatial distribution of instantaneous vorticity is extremely patchy near the crest, with isolated
filaments of high positive and negative vorticity that have steep gradients. In-between the patches and below the still water line are large expanses with nearly zero vorticity. Vorticity measurements at different phases at the same point showed that, as flow progressed, there is two-dimensional merging of co-rotating filaments and cancellation of the counter-rotating. It was observed that instantaneous vorticity filaments are created in a large range of sizes, peaking at the height of the wave used. Mean vorticity was observed to have completely different characteristics. The principal feature of the phase-averaged vorticity is a strong positive vorticity structure that appears in the flow near the shear boundary layer. A possible reason for this is that vorticity filaments have sizes distributed over several scales present in the flow. Averaging instantaneous vorticity makes small-scale vorticity filaments vanish, so the contours of the mean flow vorticity only show the development of large-scale structures within and around the shear layer, which develops and moves downstream. This occurs near the shear layer, around elevation $z = 0$, where the uprush opposes the undertow. At this point a large clockwise-rotating vorticity is formed underneath the wave crest. As the wave crest propagates along the flume, the width of the positive vorticity broadens as it sequentially interacts with downstream vorticity which are remnants of previous breaking waves. This main structure of the wave diffuses rapidly into the interior of the wave after breaking, where molecular diffusion plays a part in stretching the filaments and redistributing the vorticity. Below the trough level, mean flow vorticity is about the same order of magnitude as the wave phase speed divided by the local water depth $c/h = \sqrt{gh}$. Peak magnitudes of both instantaneous and mean flow vorticity, generated by wave breaking, are one order of magnitude greater than the phase speed divided by local depth $c/h$.

The results presented here may be useful for modeling two-dimensional turbulent flows. Identification of scaling laws for filaments may also bring some new insights. Statistical properties of the filaments, including filament life time, number of filaments per unit volume, number of high vorticity filaments impinging the bottom and their magnitudes could also be determined from this data, as this information is considered vital in sediment transport studies.

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Planning for desalination in the context of the Western Cape water supply system

C L Blersch, J A du Plessis

As water demands continue to grow, South Africa is starting to consider seawater desalination as a potential future supply source, and it is currently being investigated at a feasibility level in a number of coastal cities, including Cape Town. Desalination is different to conventional surface and groundwater supply sources in that it is climate-resilient, having an assurance of supply of essentially 100 percent. However, the increased reliability comes at a great cost. This paper presents a methodology developed for modelling a proposed desalination plant as an integrated component of the Western Cape Water Supply System, in order to optimise system operating rules and cost. The modelling entailed short-term and long-term system analyses in the Water Resources Yield Model and Water Resources Planning Model, and estimation of first-order capital and operating costs in order to calculate and compare Unit Reference Values. The maximum increase in yield was found to occur when the seawater desalination plant is used as a base supply, operational all the time. There was little benefit, in terms of system yield, in using the desalination plant as an emergency supply source only. Unit reference values for the desalination plant decrease as the percentage supply from the plant increases, meaning that the lowest possible cost per cubic metre of water supplied is when the desalination plant is used as a base supply. It was also apparent that the unit reference values decrease with an increase in desalination plant capacity, suggesting that, from an economic perspective, the optimal solution would be to have one large desalination plant operational immediately.

INTRODUCTION

Desalination can take the form of seawater desalination, desalination of brackish or polluted groundwater and the use of membrane processes in advanced water treatment processes for water reuse. All of these forms of desalination are relevant in South Africa and are currently being considered as possible supply sources for many major cities, including Cape Town. However, the focus of this paper is seawater desalination only, firstly because it is likely to be a large-scale supply source (as opposed to brackish groundwater desalination which will be on a much smaller scale), and secondly because the practicality and acceptance of reuse of treated effluent has not yet been fully researched in South Africa. Further limitations are discussed in the conclusion of the paper.

Historically, less costly surface and groundwater resources have been available to meet South Africa’s water demands, meaning that seawater desalination has not been considered a viable water source. There are currently only six small desalination plants in operation in the country, half of which were constructed as emergency schemes in response to the severe Southern Cape drought of 2009/2010. However, as growing water demands place pressure on water resources, desalination has begun to gain traction as a viable alternative. Seawater desalination is currently being investigated at a feasibility level in Saldanha Bay, Cape Town, Port Elizabeth and Durban, and is mentioned in all of the national water planning documents (Department of Water Affairs and Forestry 2008:26; Department of Water Affairs 2010:7; Department of Water Affairs 2013b:31).

Considering the existing plants, there have been some limitations in terms of integrated planning for seawater desalination in South Africa. In Mossel Bay, for example, a 15 Mℓ/day seawater desalination plant was constructed in 2011 in response to the severe drought in the area. Since the commissioning of the plant, it has remained virtually untapped, with the municipality preferring to make use of its less expensive surface water resources (Mallory et al 2013). Similar problems have been experienced in Australia, where large-scale seawater desalination was implemented, at great cost, in response to a decade-long drought (Onishi 2010).

A feasibility study for large-scale seawater desalination for Cape Town is currently under way. The decision regarding when to proceed with seawater desalination will most likely be driven by the feasibility study and Western Cape Water Supply System (WCWSS) planning mechanisms. However,
seawater desalination differs from the surface and groundwater resources of the WCWSS, firstly in its higher capital and operating costs, and secondly in its resilience to climate variability, having an assurance of supply of essentially 100 percent. Factors such as integration into the WCWSS, supply risks and cost implications may therefore be more relevant in evaluating desalination than a 98 percent assurance of supply from traditional water supply options, as is currently the case. The higher costs and assurance of supply of desalination will also impact on the overall operational philosophy.

The aim of this research was to determine what operating rules and modified planning criteria are required to optimise the implementation of large-scale desalination as part of the WCWSS, in order to ensure cost-effective provision of water at an appropriate assurance of supply.

BACKGROUND AND LITERATURE REVIEW

Existing Western Cape Water Supply System

The Western Cape area is characterised by a Mediterranean climate, with rainfall occurring in the winter months (May to October) when demands are low, and minimal rainfall occurring in the summer months when demands are high. Approximately 50 percent of the total system storage is available for storing winter flows to meet summer demand, with the remaining 50 percent required for long-term drought storage (Shand & Sparks 2004:3). The main dams of the WCWSS are operated as an integral system in order to reduce the probability of spillage, hence increasing the overall system yield (Department of Water Affairs 2007:31).

The WCWSS supplies water to various towns and irrigators in the Western Cape, including the City of Cape Town. The latest WCWSS planning documents give the total water usage from the system as 503 million m³/a in 2013 (Department of Water Affairs 2013a:21) and the total integrated system yield at a 98 percent assurance of supply as 596 million m³/a (Department of Water Affairs 2011:7). To manage demands, yield and planning models of the WCWSS have been set up in the Water Resources Yield Model (WRYM) and Water Resources Planning Model (WRPM) respectively in order to simulate the inflows and demands on the system and plot storage trajectories of the coming year. These models (WRYM and WRPM) are run annually in November in order to determine whether the dams are full enough to meet the projected summer demands at the required assurance of supply, or whether water restrictions should be implemented (Shand & Sparks 2004:6).

Seawater desalination as possible future supply source for the WCWSS was first assessed at a pre-feasibility level in 2003 as part of the Bulk Water Supply Study. A feasibility study was commissioned in 2011, and was set for completion at the end of 2013, although the findings have not yet been released. The anticipated capacity based on the feasibility study would be 150 Mℓ/day, upgradeable to 450 Mℓ/day (Department of Water Affairs 2013a:12).

Local research into desalination and integrated system modelling

Despite the various feasibility studies which are currently under way, and the ambitious outlook for seawater desalination development in South Africa, there has been little research into the conjunctive use of seawater desalination with surface water supply sources, and how to integrate seawater desalination plants into existing water supply systems. The only South African authors who have presented any material on the topic are Mallory et al (2013) who undertook a study into the optimal operation of the seawater desalination plant in Mossel Bay using the Water Resources Modelling Platform. The work of Mallory et al (2013) began to tackle some of the key questions associated with the value of water to different users, and setting of operating rules for a desalination plant.

Although not specifically related to seawater desalination, the work of Van Niekerk and Du Plessis (2013a:543) into the conjunctive evaluation of the costs and benefits of inter-basin water transfer schemes illustrates a number of useful principles which could be easily applied to integrate system modelling for seawater desalination. Van Niekerk and Du Plessis (2013a:543) found that the so-called incremental approach has historically been followed in evaluating the viability of inter-basin transfer schemes, which assumes that volume transferred per annum is equal to the difference between the projected future water demands and the current system yield. The authors found that the original projections vastly exceeded the actual transfers, mainly because the incremental approach ignores the stochastic nature of the conditions of the receiving system. To address these concerns, Van Niekerk and Du Plessis (2013a:543) propose a comprehensive approach in which the receiving water basin and the transfer scheme are modelled stochastically as an integrated system in the WRPM. Taking this concept further, Van Niekerk and Du Plessis (2013b:551) postulate that the results of such an analysis should be used when calculating the Unit Reference Value (URV) of any newly proposed scheme (Van Niekerk & Du Plessis 2013b:552).

Estimating desalination costs

South African approach to desalination costing

Du Plessis et al (2006) present a step-by-step guide to determining first-order capital and operating costs for desalination plants of different sizes in South Africa. The process centres on selecting a plant capacity, determining the required membrane area and feed water pressures, and calculating capital and operating costs with these parameters as a base. In terms of capital costs, the desalination cost component can be estimated as a function of the membrane area, the pretreatment costs can be calculated as a function of the feed flow rate, and the combined cost of the intake, outlet and post-treatment systems can be estimated as a percentage of the capital cost of the desalination plant and pre-treatment system (15 to 30 percent).

Du Plessis et al (2006) suggest that the operating costs comprise energy costs, which can be estimated based on the total energy requirement of the plant (which is primarily a function of feed-water pressure), chemical costs of approximately R0.50/m³, membrane replacement costs based on a unit cost of R165/m² of membrane area and a membrane lifespan of six years, and annual labour and maintenance costs of about 5 percent of the total capital cost. All costs and rates provided by Du Plessis et al (2006) are for a base date of 2006 and were escalated to 2014 at 6 percent per annum.

This methodology was applied to the proposed Cape Town desalination plant, situated on the west coast, based on a capacity of 100 Mℓ/day and a two-pass system, giving a first-order (2014) estimate of the capital cost of R1 054 million and the total specific cost of R16.44/m³.

International seawater desalination literature

In terms of international literature on seawater desalination cost estimation, Karagiannis and Soldatos (2008) studied almost 100 different seawater desalination plants and presented a summary of first-order desalination costs derived from literature. Based on their costs, a 100 Mℓ/day seawater desalination plant would cost approximately R9.84/m³. Similarly, Witholz et al (2008) collated a database of over 300 seawater desalination plants and derived an equation for estimating capital costs for seawater reverse osmosis desalination plants based on the plant
capacity. Using their equation, a 100 Mℓ/day seawater desalination plant would have an equivalent 2014 cost of R1 862 million and unit production cost of R9.95/m³. In a more recent study, Ghaffour et al. (2013) provided a review of the economics of desalination worldwide, noting that unit water costs for seawater reverse osmosis desalination have decreased rapidly over the past decade, meaning that a 100 Mℓ/day plant could cost as little as R960 million.1

Analysis of known costs of seawater desalination plants

Figure 1 shows costs in Rand per m³/day of desalination plant capacity plotted against plant capacity for the recently constructed desalination plants in Australia, and plants in Israel, Algeria and Spain, extracted from Ghaffour et al. (2013), and the Mossel Bay desalination plant, being the only large desalination plant in South Africa.

Attempts to derive any kind of relationship from the values presented in Figure 1 were unsuccessful, given the large scatter. One of the problems with desalination costs quoted in literature is that it is often unclear whether the costs are all-inclusive or for the desalination portion of the plant only (i.e. excluding intake and outlet structures or general civil works). This is particularly relevant for larger plants. It is clear that the Australian desalination plants were significantly more expensive than recent plants in other parts of the world, most likely a result of the emergency nature of the schemes, which invariably pushes up costs. However, for Israel, Algeria and Spain the costs are fairly similar across the range of capacities, varying from R10 700 to R14 600/m³/day of desalination plant capacity. At these rates, a 100 Mℓ/day seawater desalination plant would cost in the order of R1 290 million, which is comparable to the values derived from the methodologies found in the literature.

METODOLOGY

Integration of seawater desalination into the WCWSS models

In order to model the integration of seawater desalination into the WCWSS for the purpose of this research, an approach was required in which the desalination plant could be modelled as an integral component of the existing system. The WRYM and WRPM were selected as the most appropriate tools to achieve this. Given the long history involved in the development of the existing WCWSS models in WRYM and WRPM, it seemed logical to use them as the base for the modelling in this research. The latest versions of the WRYM and WRPM models of the WCWSS were obtained from Aurecon with the permission of the Department of Water and Sanitation (DWS)².

Operating scenarios

After studying the existing operational philosophy of the WCWSS, four possible scenarios were developed for the implementation and integration of a seawater desalination plant:

- **Scenario A**: Base scenario with current system and no desalination plant, representing the status quo or base case for comparison purposes.
- **Scenario B**: Desalination plant used as a base load supply, always operational regardless of the conditions in the rest of the supply system.
- **Scenario C**: Desalination plant used as an emergency supply, only operational when the dams reach a certain threshold. Considering the penalty structure zones of the main reservoirs, four threshold or “trigger” levels were selected to give a representative spread. Based on the level-capacity curves of the major reservoirs, the four trigger levels were estimated to be equivalent to 90 percent, 70 percent, 45 percent and 15 percent of the capacity of the main system reservoirs. For each of the desalination operating scenarios B to D, three different desalination plant capacities were tested, based on information extracted from the latest WCWSS SSC Progress Report (Department of Water Affairs 2013a:13), i.e. 150 Mℓ/day (54.8 million m³/a), 300 Mℓ/day (109.6 million m³/a) and 450 Mℓ/day (164.4 million m³/a).

Water Resources Yield Model (WRYM) setup

A number of options were considered and tested for modelling a seawater desalination plant in the WRYM and WRPM, including the use of “artificial natural runoff” files or a specified “inflow channel”, as defined by the different models. These options would entail the creation of an inflow sequence to represent the seawater desalination plant, with uniform flows in each month equal to the seawater desalination plant capacity. Creating the requisite summary of statistical parameters for the inflow file for the purposes of stochastic analyses, however, proved challenging and, as a result, a simpler approach was adopted in which a “multi-purpose min-max channel” was used as an inflow channel from a zero node. The capacity of the channel was constrained to the desalination plant capacity and an appropriate penalty was assigned to the channel in order to model all the selected operating scenarios.

Historic analyses were run for a total of 77 years (1928 to 2004) for a range of target drafts in order to determine the historic firm yield, and long-term stochastic analyses were run for a total of 51 sequences to calculate the long-term assurance of supply. The number of sequences were selected based on the suggestion of Basset et al. (1994:34) that reasonable results in a long-term analysis can be obtained with at least 40 stochastic sequences, as well as consideration of other WCWSS planning studies.
Short-term stochastic analyses were also performed in the WRYM in order to create short-term yield curves for use in the WRPM. For short-term analyses, Basson et al (1994:34) suggest that at least five times the number of sequences used in a long-term analysis are required, hence at least 255 in this case. The most recent WRPM studies carried out by Aurecon use 401 sequences. Given that the computational time is much shorter for a short-term analysis, 401 sequences seemed reasonable, and were therefore used.

Water Resources Planning Model (WRPM) setup
The WRPM setup was modified in the same way as for the WRYM. The base model setup had only ten years of growth information, starting in 2013, for the 12 master control channels. The growth factors were extended to cover a period of 20 years, based on the growth calculation spreadsheets prepared by Aurecon as part of a recent DWS study into operating rules for the WCWSS. The growth scenario assumes an increase in urban demands of 3 percent per annum, and that only 80 percent of the anticipated water conservation and demand management savings would be achieved. The projected demands for each master control channel are shown in Figure 2.

The WRPM was run for a period of 20 years starting in 2013. A total of 401 stochastic sequences were analysed in order to match the number of sequences used in deriving the short-term yield curves.

Cost estimation and calculation of URVs
In addition to comparing the yields and supply from the WCWSS with the introduction of seawater desalination, it was considered worthwhile to compare costs of the selected operating scenarios. Based on a review of the available costing methodologies, the South African approach followed by Du Plessis et al (2006) was selected as most fitting, given that it is a local methodology, uses membrane area rather than plant capacity alone to estimate capital costs, is simple to understand and apply, and for the test plant of 100 Mℓ/day compares well in terms of calculated costs to the methods considered.

The methodology of Du Plessis et al (2006) can be used for estimating capital and operating costs, but the methodology as is only gives unit costs of water based on average plant capacity. In order to provide costs which can be used to compare the operating scenarios, it was considered more appropriate to calculate URVs. The URV approach, as presented by Hoffman and Du Plessis (2008), was adapted by Van Niekerk and Du Plessis (2013a) by using the actual volume of water supplied based on a stochastic analysis in the WRPM to calculate the URVs of inter-basin transfers. This approach was applied to the WCWSS by using the modelled annual volumes of water supplied from the desalination plant, as extracted from the WRPM analyses, to calculate costs. The following adaptations were made to the costing approach of Du Plessis et al (2006) in order to provide URVs for the modelled scenarios:

- Capital costs were determined based on the desalination plant capacity and escalated to the start date of the analysis. For the same desalination plant capacity, the capital costs for all scenarios were the same.
- The total energy consumption of the plant was calculated based on its design capacity, and then these costs were factored to calculate the annual energy costs based on the annual volume of water supplied, as derived from the WRPM analyses.
It was assumed that the membranes would be replaced every six years for a plant operating at full capacity. The membrane life was increased to up to 12 years, depending on the actual annual output from the plant as a percentage of its capacity.

Maintenance and labour costs were calculated as a function of the capital costs, and were adjusted depending on the annual output of the plant.

Chemical costs were calculated by multiplying a specific cost of chemicals by the actual desalination plant supply per annum. The capital, operating and maintenance costs were summed per annum ($C_n$), and the NPV determined over the analysis period ($n$) of 20 years for a discount rate ($r$) of 8 percent. Similarly, the NPV of the water supplied from the desalination plant ($W_n$) was determined, and hence, the URV (in R/m³) calculated for each scenario based on Equation 1.

$$ URV = \frac{\sum(C_n)}{(1 + r)^n} - \frac{\sum(W_n)}{(1 + r)^n} $$  (1)

As in the approach of Van Niekerk and Du Plessis (2013a), cognisance was taken of the stochastic variation in the supply and the resulting stochastic variation of the URVs. The cost components which are a function of the actual desalination plant output, and hence vary stochastically, include the energy consumption, chemical costs, maintenance costs and membrane lifespan.

Note that all URVs calculated as part of this research were for the desalination plant and its associated infrastructure only, and not for the WCWSS as a whole.

**PRESENTATION AND DISCUSSION OF RESULTS**

**System yield**

**Base scenario** For Scenario A (the base scenario with no desalination plant) the historic firm yield of the system was calculated as 530 million m³/a. Based on the stochastic analyses, this corresponds to an assurance of supply of approximately 70 percent, or 1 in 215 years. The 1 in 50, 1 in 100 and 1 in 200 year yields of the system were calculated as 580 million m³/a, 553 million m³/a and 532 million m³/a respectively. The results for Scenarios B to D were compared to the base scenario results in order to determine the increase in yield resulting from the addition of a desalination plant to the system.

**Historic firm yield** Figure 3 shows the increase in the historic firm yield compared to the base scenario for all the seawater desalination plant operating scenarios that were analysed.

For Scenario B, with the desalination plant operational 100 percent of the time, the increase in historic firm yield is very close to the desalination plant capacity (as expected) for all three capacities. For Scenario C, with the desalination plant operational only when the dams are not spilling, the results are almost identical, which suggests that reducing the desalination plant output when the dams are spilling has little impact on the system yield. Considering Scenario D, as the reservoir trigger level at which the seawater desalination plant kicks in is lowered, the increase in historic firm yield decreases substantially. This trend is more clearly viewed in a plot of the increase in historic firm yield of the system against the desalination plant trigger level, as presented in Figure 4.

The results show a clear logarithmic pattern. Logarithmic trend lines plotted for each desalination plant capacity showed a good correlation with the modelled data. The coefficients of the logarithmic equations for each curve appeared to be related to the desalination plant capacity, and were therefore normalised based on the plant capacity and averaged to provide the generic equation as shown in Equation 2. This equation could be used to estimate the increase in historic
firm yield in million m$^3$/a ($\Delta HFY$) of the WCWSS for any desalination plant capacity and reservoir trigger level. The average annual desalination plant capacity ($Q_a$) is expressed in million m$^3$/a, and the desalination plant trigger level ($T_r$) as a percentage of the system storage in the main reservoirs.

$$\Delta HFY = 0.425Q_a \ln T_r + 0.965Q_a$$

(2)

The logarithmic shape suggests that increasing the dam trigger level from say 20 percent to 30 percent, will have a significantly greater impact on the historic firm yield than increasing the trigger level from say 80 percent to 90 percent. It also shows that there is no “turning point” or optimal trigger level. In other words, the maximum increase in historic firm yield is achieved when the desalination plant is always operational.

**Stochastic results**

Figure 5 shows the increase in 1 in 50 year yield from the base scenario, based on the results from the stochastic analyses.

The trends across the trigger levels and capacities are very similar to the historic firm yield results, showing a clear logarithmic pattern, and logarithmic trend lines fitted to the data show a good correlation. As for the historic firm yield results, the logarithmic equation coefficients were normalised based on the desalination plant capacity, and averaged, giving the generic Equation 3 for calculating the 1 in 50 year yield in million m$^3$/a ($\Delta SY$) for any desalination plant capacity and trigger level.

$$\Delta SY_{50} = 0.393Q_a \ln T_r + 1.003Q_a$$

(3)

Comparable plots prepared for the 1 in 100 year and 1 in 200 year yields showed similar trends, suggesting that the introduction of the desalination plant merely shifts the yield-reliability curve up without changing its shape. A summary of the equations derived for estimating the increase in yield for the three return periods is presented in Table 1.

**Table 1 Equations derived for estimating the increase in yield of the WCWSS for any desalination plant capacity and reservoir trigger level**

<table>
<thead>
<tr>
<th>Return period of yield (years)</th>
<th>Equation for estimating increase in yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in 50</td>
<td>$\Delta SY_{50} = 0.393Q_a \ln T_r + 1.003Q_a$</td>
</tr>
<tr>
<td>1 in 100</td>
<td>$\Delta SY_{100} = 0.384Q_a \ln T_r + 0.996Q_a$</td>
</tr>
<tr>
<td>1 in 200</td>
<td>$\Delta SY_{200} = 0.387Q_a \ln T_r + 0.948Q_a$</td>
</tr>
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</table>

**Seawater desalination plant supply**

**Median supply based on WRPM analyses**

Figure 6 illustrates the supply from the seawater desalination plant as a percentage of its capacity in each year (2013 to 2032), for all scenarios, based on a 150 Mℓ/ day plant. The values shown were derived from the WRPM analyses and are based on the median annual supply of the 401 stochastic sequences that were analysed.

For Scenario B (as expected) the seawater desalination plant would always be 100 percent operational. For Scenario C the seawater desalination plant would start out at 90 percent of its capacity, increasing
to 100 percent by 2015. Lowering the trigger level (Scenario D), lowers the output from the seawater desalination plant as a percentage of its capacity. For the lowest trigger levels of 45 percent and 15 percent, the seawater desalination plant would not be operational for the entire analysis period. Similar plots prepared for 300 Mℓ/day and 450 Mℓ/day plants show that, as the seawater desalination plant capacity increases, the supply from the plant as a percentage of its capacity decreases.

**Stochastic variability in supply**

The results presented in Figure 6 are based on median values of the 401 stochastic sequences analysed, and were selected as representative values for calculating URVs. In order to illustrate the possible stochastic variability in supply from the desalination plant, the 5th and 95th percentile annual values were plotted for Scenario C and for Scenario D with a 70 percent trigger level, as shown in Figure 7.

For Scenario C the maximum stochastic range varies between -28% and +9% from the median, decreasing to zero by the end of the analysis period (2028), when even the wetter sequences required the desalination plant to be fully operational. For Scenario D, with a 70 percent trigger level, the stochastic range is much greater, varying between -25% and +50% from the median, and hence making the choice of what values to use as input into the URV calculations more critical.

**Comparison with traditional approach**

The research of Van Niekerk and Du Plessis (2013b:543) showed that the annual supply from a new water supply source (in their case an inter-basin transfer scheme) is generally estimated by calculating the deficit between the projected annual demands of the system and the existing system capacity, taking any limitations in the capacity of the new water supply source into consideration. Applying this principle, using the demand projections as presented in Figure 2 and the 1 in 50 year yield of 580 million m³/a, the annual system deficit was calculated. Assuming that a new desalination plant would have to meet this deficit, for the selected capacities of 150 Mℓ/day, 300 Mℓ/day or 450 Mℓ/day, the annual supply as a percentage of the desalination plant capacity was calculated, as shown in Figure 8.

Based on the traditional approach, a 150 Mℓ/day seawater desalination plant would operate at 18 percent of its capacity in 2013, increasing to 100 percent by 2018. Comparing these values to the WRPM results in Figure 6, the traditional approach and the WRPM approach provide vastly different values in terms of supply from the desalination plant, particularly for the lower reservoir trigger levels. This will impact on the calculated URVs.

**Planning for future interventions**

**Traditional approach**

The typical approach that would be followed in determining when a future intervention is required in a water supply system would be a water balance of system demands and supply. The water demands that were projected as part of this study, based on the recent work by Aurecon (presented in Figure 2), were plotted along with the calculated 1 in 50 year base yield of 580 million m³/a (Scenario A) and the 1 in 50 year yields with a seawater desalination plant as derived from the WRYM analysis (Scenario B), as shown in Figure 9. Based on this approach, a 150 Mℓ/day seawater desalination plant would meet the system demands until 2017, a 300 Mℓ/day or 450 Mℓ/day, the annual supply as a percentage of the desalination plant capacity was calculated, as shown in Figure 8.
Base scenario using the WRPM

In the WCWSS analysis in the WRPM and part of the recent study on the development of integrated annual and real time operating rules for the WCWSS, revised water restriction levels were developed. The scenario which Aurecon found to be the most realistic is provided in Table 2, and was used in the WRPM analyses. As shown in Table 2, Level 1 curtailments cannot be applied more than once in every 20 years, Level 2 curtailments no more than once in every 100 years and Level 3 curtailments no more than once in every 200 years. Stricter curtailments are applied to agricultural demands than domestic demands.

Seawater desalination plant scenarios in the WRPM

Plots similar to Figure 10, showing curtailment levels, were extracted from the WRPM for the various seawater desalination operating scenarios and capacities. A summary of the critical points at which the acceptable frequency of curtailments are exceeded are provided in Table 3. The earliest dates at which an intervention is required in each case are highlighted in bold.

Considering Scenario B, the addition of a 150 Mℓ/day plant to the system would delay the date at which a new intervention is required to 2019. A plant capacity of 300 Mℓ/day would provide a further two years to...

Figure 10 shows a box-and-whisker plot of the subsystem curtailment for the base scenario (Scenario A) from 2013 to 2032 as derived from the WRPM analysis. The critical lines or whiskers which correspond to 1 in 20 year, 1 in 100 year and 1 in 200 year curtailment levels are labelled.

Curtailment Level 1 would be reached (approximately) at a frequency of 1 in 20 years by 2018. Curtailment Level 2 would be reached at a frequency of 1 in 100 years by 2016, and curtailment Level 3 would be reached at an assurance of 1 in 200 years by 2016. These critical points are marked with circles. Considering these three trigger years, the earliest that a new water supply intervention would be required for the WCWSS in order to ensure that the frequency of curtailments stays within acceptable limits would be 2016.

Table 2 Level of restrictions used in WRPM analyses

<table>
<thead>
<tr>
<th>Level of curtailment</th>
<th>Acceptable frequency of restrictions</th>
<th>Restricted water demand as a percentage of normal demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 in 10 years</td>
<td>Domestic: 100% Agricultural: 100%</td>
</tr>
<tr>
<td>1</td>
<td>1 in 20 years</td>
<td>Domestic: 93% Agricultural: 75%</td>
</tr>
<tr>
<td>2</td>
<td>1 in 100 years</td>
<td>Domestic: 85% Agricultural: 50%</td>
</tr>
<tr>
<td>3</td>
<td>1 in 200 years</td>
<td>Domestic: 71% Agricultural: 20%</td>
</tr>
</tbody>
</table>

Table 3 Year at which curtailment level would be reached for all scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>150 Mℓ/day plant</th>
<th>300 Mℓ/day plant</th>
<th>450 Mℓ/day plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in 20 year</td>
<td>1 in 100 year</td>
<td>1 in 200 year</td>
<td>1 in 100 year</td>
</tr>
<tr>
<td>B</td>
<td>2021</td>
<td>2019</td>
<td>2019</td>
</tr>
<tr>
<td>C</td>
<td>2021</td>
<td>2019</td>
<td>2019</td>
</tr>
<tr>
<td>D (90%)</td>
<td>2021</td>
<td>2019</td>
<td>2019</td>
</tr>
<tr>
<td>D (70%)</td>
<td>2021</td>
<td>2018</td>
<td>2019</td>
</tr>
</tbody>
</table>


No desalination plant
2021, and a capacity of 450 Mℓ/day, an additional two years to 2023. For Scenario C and Scenario D with a 90 percent trigger level, the results are identical to those of Scenario B, suggesting that the decision regarding whether to operate the seawater desalination plant 100 percent of the time or only when the dams are not spilling would not be influenced by curtailment requirements.

As the trigger levels decrease, the benefit of the seawater desalination plant in terms of delaying the requirement for a new scheme is reduced. For the lowest trigger levels the seawater desalination plant provides almost no benefit in terms of reducing curtailments, suggesting that there is very little benefit in using the desalination plant as an emergency water supply source.

Unlimited desalination plant capacity

The results are derived from assumed seawater desalination plant capacities of 150, 300 and 450 Mℓ/day. The reverse of this approach would be to ask: “What desalination plant capacity is required in order to meet future demands for the next 10 or 20 years?” Based on the traditional approach, a 400 Mℓ/day seawater desalination plant would meet the system requirements up to 2023, and a 1 000 Mℓ/day plant would meet the requirements up to 2033. The WRPM results confirm that, with a 450 Mℓ/day desalination plant, curtailments would be kept within acceptable limits until 2023. However, from the WRPM results it is not possible to confirm what capacity is required to meet the demands beyond this point.

The WRPM analyses were repeated for Scenarios C and D (with a 70 percent trigger level only) with a seawater desalination plant of unlimited capacity. The monthly supply from the seawater desalination plant over the analysis period was determined, based on the median values of the 401 stochastic sequences. The maximum monthly supply for Scenario C would be 2 100 Mℓ/day in 2023, increasing to 2 900 Mℓ/day in 2033.

2023, increasing to 2 900 Mℓ/day in 2033.

Table 4. URVs for all scenarios and seawater desalination plant capacities derived from WRPM analyses

<table>
<thead>
<tr>
<th>Scenario</th>
<th>150 Mℓ/day plant</th>
<th>300 Mℓ/day plant</th>
<th>450 Mℓ/day plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5th percentile</td>
<td>95th percentile</td>
<td>5th percentile</td>
</tr>
<tr>
<td>D (70%)</td>
<td>R11.16</td>
<td>R15.49</td>
<td>R22.77</td>
</tr>
<tr>
<td>D (45%)</td>
<td>R37.06</td>
<td>R99.00*</td>
<td>R99.00*</td>
</tr>
<tr>
<td>D (15%)</td>
<td>R54.68</td>
<td>R99.00*</td>
<td>R99.00*</td>
</tr>
</tbody>
</table>

* URVs in excess of R99/m³ were capped at R99/m³ as a representative upper limit.

For Scenario D the plant would have to supply up to 1 500 Mℓ/day in 2023, increasing to 2 100 Mℓ/day by 2033.

The problem with adopting this approach is that it does not allow for any failures over the analysis period, i.e. for Scenario D with a 70 percent trigger level, a 1 500 Mℓ/day plant would meet the demands until 2023 with a 100 percent assurance of supply. This differs from the normal risk allowance that would be applied in planning for a water supply system in South Africa.

Comparison of costs

Traditional approach to calculating URVs

The research of Van Niekerk and Du Plessis (2013a:551) showed that the traditional approach to calculating URVs uses the water supply from a scheme calculated based on a water balance between annual demands and a 1 in 50 year system yield. As a starting point, this traditional approach was applied for calculating the URV of the seawater desalination plant. For a discount rate of 8 percent, the traditional approach yielded URVs of R11.35/m³ for a 150 Mℓ/day seawater desalination plant, R11.46/m³ for a 300 Mℓ/day seawater desalination plant and R11.48/m³ for a 450 Mℓ/day seawater desalination plant.

WRPM-based approach to calculating URVs

In the WRPM-based approach, as applied for this research, capital and operating costs were calculated for each scenario using the annual supply from the seawater desalination plant as derived from the WRPM analyses. Using these costs and the associated seawater desalination plant supply, URVs were calculated for each scenario and seawater desalination plant capacity as presented in Table 4. The values highlighted in bold are the median values (50th percentile) which were considered to be the most reasonable for the purposes of comparison between the scenarios and with other possible interventions. The 5th and 95th percentile URVs are also shown in order to provide an indication of the possible stochastic range of the costs.

Considering the median (50th percentile) values in Table 4, it would appear that, for all capacities, the URV increases as the trigger level decreases. It therefore seems that operating the seawater desalination plant as an emergency type scheme, as exemplified by the 45 percent and 15 percent trigger levels, would be expensive. Although the annual operating costs would be much lower for an emergency scheme, the initial capital cost would be the same regardless of the actual seawater desalination plant output. The results suggest that this initial capital cost outweighs any reduction in operating costs when the seawater desalination plant output is reduced.

For a trigger level of 70 percent, the URV for a 150 Mℓ/day plant is much more reasonable at R15.49/m³, decreasing to R12.21/m³ for a 90 percent trigger level. For Scenario B and Scenario C the desalination plant would be fully operational once constructed, giving a URV of R10.96/m³. Based on these values it would appear that the lowest possible URV occurs when the desalination plant is 100 percent operational.

Comparing the three capacities that were analysed, the URVs decrease slightly with an increase in seawater desalination plant capacity. This suggests that it would be more beneficial from an economic perspective to construct a larger seawater desalination plant now, rather than phasing it in smaller lanes.

Comparing the results from the traditional approach to the WRPM-based approach, the results for a 150 Mℓ/day plant are actually similar to Scenario B/C. This suggests that, although the annual supply from the seawater desalination plant differs vastly between the traditional and WRPM approaches in the initial years, both approaches suggest that the seawater desalination plant would be 100 percent...
The increase in system yield as a result of the desalination plant appears to be logarithmically related to the reservoir trigger level. Generic equations were developed which could be used to estimate the increase in historic firm yield and 1 in 50 year yield of the WCWSS for any seawater desalination plant capacity and reservoir trigger level.

The maximum increase in yield occurs when the seawater desalination plant is used as a base supply, operational all the time. The increase in yield decreases with a decrease in the reservoir trigger level, meaning that there is little benefit in using the seawater desalination plant as an emergency supply source.

With a 150 Mℓ/day desalination plant, curtailment requirements would be kept within acceptable levels until 2019, with a 300 Mℓ/day plant until 2021 and a with a 450 Mℓ/day plant until 2023, for a desalination plant which is 100 percent operational. Using the plant as an emergency supply source would add little benefit in terms of limiting curtailment requirements.

URVs for the seawater desalination plant decrease as the percentage of time for which the seawater desalination plant is operational increases. This means that the lowest possible cost per cubic metre of water supplied is when the seawater desalination plant is operational 100 percent of the time (R10.96/m³ for a 150 Mℓ/day plant).

It would also appear that the URVs decrease with an increase in seawater desalination plant capacity, meaning that constructing a larger seawater desalination plant upfront may be more cost-effective. However, it would have to be constructed in small enough lanes to enable restriction of the output when the total supply is not required, and cognizance would have to be taken of other relevant factors, such as availability of other schemes already in place and growth in water demands.

The lower the reservoir trigger level, the larger the stochastic variation in the supply from the seawater desalination plant, and hence the larger the variation in the URVs. Hence, for a seawater desalination plant operational as a base supply source, undertaking stochastic modelling to calculate URVs is less important than for an emergency supply source.

CONCLUSIONS

Through the current WCWSS planning processes, seawater desalination has been earmarked as a potential future supply source for the area. In order to maximise the benefits and minimise the costs of a seawater desalination plant, it needs to be considered as an integral part of the current system. In order to undertake integrated system analysis, the WRYM and WRPM were selected as the most appropriate tools. Short-term and long-term analyses were completed for a variety of possible seawater desalination plant operating scenarios and capacities in order to determine the increase in system yield and the annual supply from the seawater desalination plant. First-order capital and operating costs were estimated using the South African methodology of Du Plessis et al (2006). Combined with the annual supply values from the WRPM, these costs were used to calculate and compare the URVs of the desalinated water.

The following conclusions can be drawn from the results of the modelling and cost analyses:

- It is possible to model a desalination plant as an integral part of a surface water supply system. With the appropriate costing tools, a similar analysis approach could be applied to other non-conventional resources, such as water reuse.
- The increase in system yield as a result of the desalination plant appears to be logarithmically related to the reservoir

NOTES

1. The following exchange rates were issued in the costing analyses and comparison of known costs (South African Reserve Bank 2014):
   - 1 Australian Dollar (AUS$) is equivalent to R9.80
   - 1 United States Dollar (US$) is equivalent to R10.70
   - 1 Euro (€) is equivalent to R14.70

2. DWS (Department of Water and Sanitation) was previously the Department of Water Affairs and Forestry (prior to 2009) and subsequently the Department of Water Affairs (from 2009 to 2014). In this paper, the department is referred to by its current designation (DWS), apart from in the references, where the name at the time of publication of the referenced document is used.

REFERENCES


Shand, M & Sparks, A 2004. Optimisation of system operating rules for the Western Cape Water Supply System. s.l.s.n.


Numerical modelling of flexible pavement incorporating cross-anisotropic material properties

Part I: Surface circular loading

J.W. Maina, F. Kawana, K. Matsui

Accurate numerical modelling of the behaviour of road pavement layers is an important requirement for the design and evaluation of road pavements. This modelling includes the prediction of pavement performance under the action of traffic loading and environmental factors. Depending on the complexity of the models, properties of pavement layers that may be considered are wide-ranging — from wide- or nonlinear elastic to cross-anisotropic through to linear visco-elastic-plastic. Some properties, such as cross-anisotropic, are not only related to placement and compaction of the pavement layers, but are also inherent to the materials used. Other properties, such as linear visco-elastic-plastic, are specific to asphalt concrete and depend on the speed and magnitude of traffic loading, as well as the environment (temperature) in which the road is located. This paper presents basic theoretical derivation of numerical modelling of a flexible pavement considering cross-anisotropic material properties (with isotropic properties as a special case). The solutions derived in this paper are based on Hankel transformation of Navier's equations. The accuracy and validity of the solutions are verified through comparisons with a proprietary finite element method (FEM) package. For this purpose, a pavement structure composed of five main layers constituted by isotropic and cross-anisotropic (also known as transversely isotropic) material properties is analysed. In order to vary some of the layer properties with depth, the main layers were sub-layered, resulting in a 17-layer pavement system.

BACKGROUND INFORMATION

To support the movement of people and goods, as well as access to education and training, employment and health care, a sound road network — which in South Africa is most often designed and evaluated using the South African Pavement Design Method (SAPDM) — plays a key role in the socio-economic development of a country. South Africa, in the recent past, has experienced considerable growth in both passenger and freight traffic volumes because of increased economic activities. The high level of truck traffic is causing premature failures, mostly on provincial and municipal road networks.

The South African National Roads Agency Ltd (SANRAL) teamed up with the roads industry, the Council for Scientific and Industrial Research (CSIR) and academia in an effort to improve the SAPDM and the design of roads in South Africa. The partnership called for skills development and the building of capacity to optimise design, maintenance and repair strategies against road (premature) failures. To achieve this, a better understanding of the properties of materials used in road constructions and modelling of structural behaviour of pavements was deemed necessary.

The SAPDM, which uses multilayer linear elastic (MLE) theory to determine pavement structural responses, was introduced in the late 1960s and early 1970s. Since its introduction, developments in material characterisations and numerical modelling have taken place. In addition, advanced computer software and hardware make it possible now to determine the stress/strain/displacement distributions of pavements under surface loading in a matter of seconds.

This paper presents the outcome of the efforts to improve the standard pavement analysis for circular surface loading by introducing capability to consider cross-anisotropic material properties (with isotropic as a special case). This capability is not possible in the current SAPDM software. After rigorous validation and verification, software resulting from this development has now become the analysis engine of the new SAPDM, and it will be used when the...
loading on the surface of the road pavement is circular in shape.

**MOTIVATION FOR INCORPORATING CROSS-ANISOTROPIC PROPERTIES**

Cross-anisotropy is the most prevalent but less considered type of materials properties in pavement layers. In this regard, the elastic properties in the lateral and vertical directions are considered to be different. The road construction process involves directional rolling compaction of materials, which invariably results in different mechanistic properties in vertical and horizontal directions (Masad et al 2006). A cross-anisotropic material has a vertical axis of symmetry of rotation, i.e. the elastic properties are equivalent in all directions perpendicular to the axis of symmetry (horizontal or radial direction). In general, these properties are different from those in the direction parallel to the axis (vertical direction).

In engineering mechanics, cross-anisotropy of an elastic material is defined by five independent elastic constants — two elastic moduli in vertical and horizontal directions (\(E_v\) and \(E_h\)), two Poisson’s ratios (\(v_{r\theta}\) and \(v_{r\theta}\)) and one shear modulus (\(G_{\theta\theta}\)) as presented by Love (1944). Several experiments have also proven the existence of cross-anisotropic properties in pavement layers, and these properties have been measured in the laboratory (Correia 1999; Adu-Osei et al 2001; Tutumluer et al 2003; Masad et al 2006). Typically, the level of cross-anisotropy is characterised by the ratio of the horizontal to vertical modulus (\(E_h/E_v\)). In contrast, isotropic materials have the same elastic properties in both the vertical and horizontal directions, which are described by three independent elastic constants: \(E, v\) and \(G\).

The importance of incorporating cross-anisotropic material property in the pavement analysis was demonstrated by Masad et al (2006), who showed how the accuracy of calculated pavement displacements improved when the aggregate base layer was considered to be cross-anisotropic. In their study, Masad et al (2006) used a Finite Element (FE) program to calculate pavement surface displacements, and compared their results with measured values from 246 different sections in the Association of American State Highway Officials (AASHO) road test. The study revealed that the displacement calculations using isotropic material properties tended to be smaller than measured displacements in the field. Further analysis showed that the errors between measured and calculated responses were minimised when the elastic modulus in the horizontal direction was assumed to be 30% of the elastic modulus in the vertical direction (\(E_j/E_v = 0.3\)). Also, the findings seem to be in agreement with results reported in another study on the level of anisotropy (\(E_j/E_v\)) of unbound and chemically stabilised aggregate systems (Salehi-Astbiani et al 2008).

Other researchers such as Emeriault and Chang (1997), and Masad et al (2004) have also used the micromechanics theory to determine cross-anisotropic properties of granular base materials. Adu-Osei et al (2001) used the linear elastic theory coupled with system identification to develop a testing system, in a triaxial test setup, that could identify the five parameters that represent cross-anisotropic properties of unbound materials. This test was developed in a collaborative research between the universities of Illinois and Texas, whereas the field validation data was collected from a full-scale pavement test study conducted at Georgia Tech (Georgia Institute of Technology). The validation of the anisotropic modelling approach was accomplished by analysing conventional flexible pavement test sections, where the GT-PAVE finite element program was used to predict responses to loading in an unbound aggregate base layer, and compared these predicted responses to the measured values (Tutumluer et al 2003). Wang and Liao (1999) listed and meticulously summarised the numerous studies for cross-anisotropic half-space subjected to different types of loading, such as line loads, circular loads, parabolic loads, ring loads, and varying rectangular loads.

**THEORETICAL DEVELOPMENT**

**Linear elasticity**

Because of azimuthal symmetry for circular surface loading, responses present in axisymmetric elastic problems are displacements \(u = u(r,z), w = w(r,z)\) in horizontal, \(r\)– and vertical, \(z\)–directions respectively, normal stresses \(\sigma_r, \sigma_z\) and \(\sigma_\theta\) in the horizontal, \(r\)–and vertical, \(z\)– and circular, \(\theta\)– directions respectively, as well as shear stress \(\tau_{rz}\) on the r or z face in the respective z or r direction, as shown on Figure 1.

In this regard, the differential equilibrium equations in cylindrical coordinates may be expressed using modified Navier’s Equations 1 and 2:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} = \frac{\partial u}{\partial r} (1)
\]

\[
\frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial \sigma_\theta}{\partial \theta} = \frac{\partial w}{\partial r} (2)
\]

Corresponding axisymmetric strains may be expressed in terms of axisymmetric displacements as follows:

\[
\varepsilon_r = \frac{\partial u}{\partial r} \quad \varepsilon_\theta = \frac{\partial u}{\partial \theta} \quad \gamma_{r\theta} = \frac{\partial w}{\partial r} + \frac{\partial w}{\partial \theta} (3)
\]

**Generalised Hooke’s law**

In linear elasticity, if the stress is sufficiently small, Hooke’s law is used to represent the material behaviour and to relate the unknown stresses and strains. The general equation for Hooke’s law is given below:

\[
\varepsilon_{ij} = \frac{1}{E} \sigma_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} s_{ijkl} \sigma_{kl} (4)
\]

Where: \(i, j = 1, 2, 3\)

In this case \(s_{ijkl}\) is a fourth-rank tensor called elastic compliance of the material. Each subscript of \(s_{ijkl}\) takes on the values from 1 to 3, giving a total of \(3^4 = 81\) independent components in s. However, due to the symmetry of both \(s_{ijkl}\) and \(\sigma_{ijkl}\) the elastic compliance s must satisfy the relation:

\[
s_{ijkl} = s_{ijkl} = s_{ijk\ell} = s_{ij\ell k} \quad (5)
\]

It follows from Equation 5, therefore, that the generalised Hooke’s law presented in Equation 4 can be simplified to become:

\[
\varepsilon_i = s_{ij} \sigma_j \quad (6)
\]

Where: \(i, j = 1, 2, 3\).
This relationship reduces the number of $s$ components to 16, as seen in the following linear relation between the pseudovector forms of the strain and stress:

$$
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{12} \\
\varepsilon_{31} \\
\varepsilon_{41}
\end{bmatrix} =
\begin{bmatrix}
s_{11} \\
s_{12} \\
s_{13} \\
s_{14}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{12} \\
\sigma_{31} \\
\sigma_{41}
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
\varepsilon_{r} \\
\varepsilon_{z} \\
\gamma_{rz}
\end{bmatrix} =
\begin{bmatrix}
s_{11} \\
s_{12} \\
s_{13} \\
s_{14}
\end{bmatrix}
\begin{bmatrix}
\sigma_{r} \\
\sigma_{z} \\
\tau_{rz}
\end{bmatrix}
$$

(7)

The $s$ matrix in this form is also symmetric. It, therefore, contains only ten independent elements. The number of independent elements is obtained by counting elements in the upper right triangle of the matrix, including the diagonal elements (i.e., $1 + 2 + 3 + 4 = 10$). Furthermore, if the material exhibits symmetry in its elastic response, the number of independent components in the compliance $s$ matrix will be reduced even further.

For example, in the simplest case of isotropic materials whose elastic moduli are the same in all directions, only three unique components $s_{11}, s_{12}, s_{44}$ exist, as shown in Equation 8. In the elastic range, these three coefficients of the ordinary model require three parameters (elastic modulus $E$, Poisson’s ratio $\nu$ and shear modulus $G$) for their definitions as follows:

$$
\begin{bmatrix}
s_{11} \\
s_{12} \\
s_{44}
\end{bmatrix} =
\begin{bmatrix}
1/\nu & -\nu/E & 0 \\
1/\nu & 1/\nu & 0 \\
0 & 0 & 1/G
\end{bmatrix}
$$

(8)

Where: $s_{11} = 1/\nu, s_{12} = -\nu/E, s_{44} = 1/G, E = \nu[2(1 + \nu)]$

In cross-anisotropic materials, however, five unique components $s_{11}, s_{12}, s_{13}, s_{33}, s_{44}$ exist, as shown in Equation 9. In the elastic range, these five components of the ordinary cross-anisotropic model require five parameters ($E_r, E_v, v_{rv}, v_{zh}, G_{hv}$) for their definitions (Lowe 1944).

$$
\begin{bmatrix}
s_{11} \\
s_{12} \\
s_{13} \\
s_{33} \\
s_{44}
\end{bmatrix} =
\begin{bmatrix}
1/\nu & -\nu/E & 0 \\
1/\nu & 1/\nu & 0 \\
0 & 0 & 1/G
\end{bmatrix}
$$

(9)

Where: $s_{11} = 1/E_r, s_{12} = -\nu_{hv}/E_r, s_{13} = -\nu_{vh}/E_v, s_{33} = 1/E_v, s_{44} = 1/G_{hv}, E_{hv} = E_v[2(1 + \nu_{vh})]$

Writing the stresses in terms of the strains would require Equation 7 to be inverted, yielding:

$$
\sigma_j = c_j \varepsilon_j
$$

(10)

Where: $c_j$ is a fourth-rank tensor called elastic stiffness of the material.

The stiffness and compliance matrices in Equation 10 are related in the following form:

$$
c = s^{-1}
$$

(11)

Three components $c_{11}, c_{12}, c_{44}$ exist for isotropic materials, whereas, in cross-anisotropic materials, five unique components $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ exist.

It should be noted that for responses to a special cross-anisotropy case where $E_r = E_v$ and $\nu_{vh} = \nu_{hv}$ exist and will be similar to the isotropic case presented by Maina and Matsui (2004).

Equation 12 below shows the five components of the elastic stiffness $c$ and Equations 13–17 define each of the five components:

$$
c =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 \\
c_{12} & c_{11} & c_{13} & 0 \\
c_{13} & c_{13} & c_{33} & 0 \\
0 & 0 & 0 & c_{44}
\end{bmatrix}
$$

(12)

Substituting Equations 12 into 10 yields:

$$
\begin{bmatrix}
\sigma_{r} \\
\sigma_{\theta} \\
\tau_{rz}
\end{bmatrix} =
\begin{bmatrix}
\sigma_{r} \\
\sigma_{\theta} \\
\tau_{rz}
\end{bmatrix}
\begin{bmatrix}
c_{11} & c_{12} & c_{13} \\
c_{12} & c_{11} & c_{13} \\
c_{13} & c_{13} & c_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{r} \\
\varepsilon_{\theta} \\
\gamma_{rz}
\end{bmatrix}
$$

(18)

Expanding Equation 18 by expressing strains in terms of elastic displacement based on Equation 3 gives:

$$
\begin{align*}
\sigma_{r} &= c_{11} \frac{\partial u}{\partial r} + c_{12} \frac{u}{r} + c_{13} \frac{\partial w}{\partial z} \\
\sigma_{\theta} &= c_{12} \frac{\partial u}{\partial r} + c_{11} \frac{u}{r} + c_{13} \frac{\partial w}{\partial z} \\
\tau_{rz} &= c_{44} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)
\end{align*}
$$

(19–21)

Substituting Equations 19–22 into Equations 1 and 2, with some rearrangement, will result in Equations 23 and 24:

$$
\begin{align*}
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{c_{44}}{c_{11}} \frac{\partial^2}{\partial z^2} \right) u &= 0 \\
\left( \frac{\partial^2}{\partial^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{c_{33}}{c_{44}} \frac{\partial^2}{\partial z^2} \right) w &= 0
\end{align*}
$$

(23–24)

Derivation of displacement and stress solutions using Hankel transformation

One convenient way of deriving displacement and stress solutions is to use Hankel transformation (Miyamoto 1977; Sneddon 1951).

In this regard, Hankel transformation of Equations 23 and 24 would yield, respectively:

$$
\begin{align*}
\left( -\xi^2 + b \frac{\partial^2}{\partial z^2} \right) \ddot{u} + a \frac{\partial^2}{\partial z^2} \ddot{w} &= 0 \\
\left( \xi^2 + d \frac{\partial^2}{\partial z^2} \right) \ddot{u} - \xi^2 \ddot{w} &= 0
\end{align*}
$$

(25–26)

Where:

$$
\begin{align*}
a &= \left( c_{11} + c_{44} \right) \frac{c_{33}}{c_{44}} \\
b &= \left( c_{13} + c_{44} \right) \frac{c_{33}}{c_{44}} \\
c &= \left( c_{11} + c_{44} \right) \\
d &= \frac{c_{33}}{c_{44}} \\
c &= \frac{c_{11} c_{44}}{c_{13}} \\
d &= \frac{c_{11} c_{44}}{c_{13}} \\
\ddot{u} &= \ddot{u}(\xi, z) = \int_{0}^{\infty} ru(r, z)f_{u}(\xi r)dr \\
\ddot{w} &= \ddot{w}(\xi, z) = \int_{0}^{\infty} rw(r, z)f_{w}(\xi r)dr
\end{align*}
$$

(27–29)

\(\ddot{u}\) and \(\ddot{w}\) are Hankel transformations of the displacements for \(u\) and \(w\), whereas \(f_{u}(\xi r)\) and \(f_{w}(\xi r)\) are Bessel functions of first kind and order 0 and 1, respectively. \(\xi\) is the parameter of Hankel transformation corresponding to \(r\). Equations 25 and 26 can be simplified by representing a differential function with respect to \(z\) as \(\lambda\). The next step is to eliminate \(\ddot{u}\) from the modified equations, and simplifying, resulting in:

$$
\left( \lambda^4 - t_1 \xi^2 \lambda^2 + t_2 \xi^4 \right) \ddot{w} = 0
$$

(30)

Where:

$$
\begin{align*}
t_1 &= \frac{b - ac + d}{bd} \\
t_2 &= \frac{1}{bd}
\end{align*}
$$

With the four roots, \(\lambda\) derived as:

$$
\lambda = \pm \sqrt{t_1 \pm \sqrt{t_1^2 - 4t_2}}
$$

(31)

Putting back the differential function in Equation 30 yields:
With reference to the roots in Equation 31, the solution form of Equation 32 depends on the sign of the coefficient: \( t_1^2 - 4t_2 > 0 \), where \( E_h > E_v \).

Furthermore, and this is very important, depending on the numerical integration methods used, it may be necessary to modify the solution for \( \tilde{w} \) to obtain stable and accurate responses of the pavement structure. The solutions presented hereunder were based on the numerical methods used in this research.

1. Solution 1: \( t_1^2 - 4t_2 > 0 \), where \( E_h > E_v \)

\[
\tilde{w} = \tilde{w}(\xi, z) = C_1(\xi)e^{\lambda_1z} + C_2(\xi)e^{-\lambda_1z} + C_3(\xi)e^{\lambda_2z} + C_4(\xi)e^{-\lambda_2z}
\]

(33)

Where:

\[
\lambda_1 = \xi \sqrt{1 - \frac{t_1^2 - 4t_2}{2}}, \quad \lambda_2 = \xi \sqrt{1 - \frac{t_1^2 - 4t_2}{2}}
\]

and \( C_1(\xi), C_2(\xi), C_3(\xi), \) and \( C_4(\xi) \) are coefficients of integration.

In order to obtain stable and accurate results of pavement responses, Equation 33 was modified to:

\[
\tilde{w} = \tilde{w}(\xi, z) = C_1(\xi) \cos(\eta z) e^{\xi_1z} + C_2(\xi) \cos(\eta z) e^{-\xi_1z} - C_3(\xi) \sinh(\eta z) e^{\eta_1z} - C_4(\xi) \sinh(\eta z) e^{-\eta_1z}
\]

(34)

Where:

\[
r_1 = \xi \sqrt{\frac{\eta_1^2 - \eta^2 - 4t_2}{2}}, \quad r_2 = \xi \sqrt{\frac{\eta_1^2 - \eta^2 - 4t_2}{2}}
\]

2. Solution 2: \( t_1^2 - 4t_2 < 0 \), where \( E_h = E_v \)

\[
\tilde{w} = \tilde{w}(\xi, z) = C_1(\xi) e^{\xi z} + C_2(\xi) e^{-\xi z} + C_3(\xi) e^{\eta z} + C_4(\xi) e^{-\eta z}
\]

(35)

| \( \sigma_1(0, \xi) \) | \( -\tilde{p}(\xi) \) |
| \( t_{1z}(0, \xi) \) | 0 |

(38)

Where:

\[
\tilde{p}(\xi) = \int_0^\infty r p(\xi) dr = \frac{p_0}{\xi} J_1(\xi a)
\]

### Pavement responses

By applying Hankel inverse transformation of all the Hankel transformed solutions, it is possible to determine solutions for pavement responses at any point in a pavement structure. However, when \( r = 0 \), the computation of \( \sigma_r \) and \( \sigma_\theta \) using Equations 19 and 20 is not straightforward because of the term \( 1/r \). The L’Hospital rule (Taylor 1952) is used to derive the solutions.

After Hankel transforms have been determined, it is possible to determine responses at any point \( (r, z) \) in the pavement structure through Hankel inverse transformation using the following equations:

#### Boundary condition – vertical circular surface loading

The boundary condition at the surface of the pavement considers an equilibrium between external and internal vertical and shear stresses. For the case shown in Figure 2, where there is only uniformly distributed vertical circular load \( P \), whose radius is \( a \) on the road surface, the Hankel transformation of the equilibrium of internal and external stresses is given as:

\[
\left\{ \begin{array}{l}
\sigma_1(0, \xi) = -\tilde{p}(\xi) \\
t_{1z}(0, \xi) = 0
\end{array} \right.
\]

(38)

Where:

\[
\tilde{p}(\xi) = \int_0^\infty r p(\xi) dr = \frac{p_0}{\xi} J_1(\xi a)
\]
\[ u(r, z) = \int_0^\infty \xi \tilde{u}(\xi, z) J_0(\xi r) d\xi \]  
\[ w(r, z) = \int_0^\infty \xi \tilde{w}(\xi, z) J_0(\xi r) d\xi \]  
\[ \sigma_r(r, z) = \int_0^\infty \xi \tilde{\sigma}_r(\xi, z) J_0(\xi r) d\xi \]  
\[ \sigma_\theta(r, z) = \int_0^\infty \xi \tilde{\sigma}_\theta(\xi, z) J_0(\xi r) d\xi \]  
\[ \tau_{rz}(r, z) = \int_0^\infty \xi \tilde{\tau}_{rz}(\xi, z) J_1(\xi r) d\xi \]  
\[ \sigma_z(r, z) = \int_0^\infty \xi \tilde{\sigma}_z(\xi, z) J_0(\xi r) d\xi \]  
\[ \sigma_{\theta c}(r, z) = \int_0^\infty \xi H_{1}(\xi, z) J_0(\xi r) d\xi \]  
\[ \sigma_{r c}(r, z) = \int_0^\infty \xi H_{2}(\xi, z) J_0(\xi r) d\xi \]  

**WORKED EXAMPLES**

Solutions developed in this study were used to compute responses at different positions within a pavement structure. The pavement structure for which the simulation results are presented here are shown in Figure 3 on page 25. A vertical load of 21.5 kN and a diameter of 238 mm resulting in a 483 kPa contact stress was used in the analysis. All layers except the asphalt layer were modelled with isotropic, linear-elastic properties. The asphalt layer was modelled with cross-anisotropic, linear elastic properties. Information on all the layers, including sub-layers, is shown in Table 1. The sub-layering of each of the upper four main layers into four sub-layers resulted in a total of 17 layers, as shown in Table 1.

The vertical stiffness in the asphalt sub-layers shows a marginal increase close to the surface that is a function of the effect of temperature variations with depth for a specific time of the day used in the analysis. The top and bottom asphalt sub-layers show significant reduction in the effective horizontal stiffness resulting from cracking initiating from both the top and bottom of the main asphalt layer.
This study

FEM

Strain, \( E_{zz} \) (MPa)

-1.0E-04
-5.0E-05
0.0E+00
6.0E-05
1.0E-04
1.5E-04
2.0E-04
2.5E-04
3.0E-04
3.5E-04
4.0E-04
4.5E-04
5.0E-04
5.5E-04

0
100
200
300
400
500
600
700
Depth (mm)

This study \( E_{zz} \) FEM \( E_{zz} \)

Discussion of results

Figures 4 and 5 show comparisons between the numerical method developed in this study and a proprietary FEM package for stress (\( S_{zz} \)) and (\( S_{ss} \)) as well as strains (\( E_{zz} \)) and (\( E_{xx} \)) distributions with depth, z-direction, under the centre of a circular surface load of magnitude 21.5 kN load (483 kPa contact stress). From these results it is clear that the closed-form solutions developed in this study have achieved a very good level of accuracy, as their results compare very well with results from a proprietary FEM package.

It is also important to mention here that the software developed from this work has become the analysis engine for the new SAPDM.

CONCLUSIONS

1. A numerical tool for the analysis of an elastic multilayer system under the action of surface circular load, considering both cross-anisotropy and isotropy material properties, has successfully been developed.
2. The numerical tool developed in this study is capable of performing analyses for an unlimited number of points in an elastic multi-layered pavement system with an unlimited number of layers, and on the surface where an unlimited number of uniformly distributed circular loads act.
3. The results shown in this paper confirm the accuracy and reliability of the closed-form theoretical solutions developed.
4. The numerical tools developed can be used to improve the design, evaluation and analysis of road/runway pavement systems.

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REFERENCES


Numerical modelling of flexible pavement incorporating cross-anisotropic material properties

Part II: Surface rectangular loading

J W Maina, F Kawana, K Matsui

In order to better understand the impact of increased loading on roads, studies on tyre-road interaction have gained prominence in recent years. Tyres form an essential interface between vehicles and road pavement surfaces. These are the only parts of the vehicle that are in contact with the road and transmit the vehicle loading to the road surface. The use of the Cartesian coordinate system is convenient in dealing with a uniform/non-uniform tyre load acting over a rectangular area, but few research reports are available that provide any form of theoretical solutions for pavement responses. This paper presents analytical solutions of responses due to rectangular loading acting on the surface of a multi-layered pavement system. The solutions developed incorporate both isotropic and cross-anisotropic material properties. The method followed is based on classical trigonometric integral and Fourier transformation of Navier’s equations. Accuracy and validity of the solutions are verified through comparisons with a proprietary finite element method (FEM) package. For this purpose, a pavement structure composed of five main layers constituted by isotropic and cross-anisotropic (also known as transversely isotropic) material properties is analysed. In order to vary some of the layer properties with depth, the main layers were sub-layered, resulting in a 17-layer pavement system.

INTRODUCTION

Motivation for this work is discussed in the companion paper by Maina et al (2017), (see pages 22–27 in this edition). Although modern era trucks transport heavier cargo, they are using relatively fewer tyres than their predecessors, and as a result they are purported to be exerting much higher contact stresses on the road surface. A good understanding of tyre-road contact stresses, and the ability to model the macroscopic behaviour of materials when subjected to varying traffic loading and environmental conditions, is therefore important for better road pavement designs and improved pavement performance. Tyres are the only part of the vehicle that are in contact with the road, and transmit the vehicle loading to the road surface through a very small contact area, called the “contact patch” or “tyre footprint”. Generally, there are two main types of truck tyres widely used on our roads – the single (or so-called wide-base tyre) and the conventional dual-tyre. A single wide-base tyre is a proportionately larger and more robust tyre that is now being used on trucks for heavy cargo. This type of tyre is expected to replace dual-tyres in the future, on condition of minimal damage to the existing road infrastructure. To be able to carry the same load as the dual-tyres, the wide-base tyre may have a much greater tyre inflation pressure and a larger individual “footprint” (but could also be smaller than the two combined “footprints” from standard dual-tyres). Research done by De Beer (2008) on tyre-pavement contact stresses has also shown tyre-pavement contact stresses to be, although dependent on the loading magnitude and inflation pressure, mostly rectangular and occasionally circular in shape. Development of solutions for circular surface loading has already been presented in the companion paper published in this edition (Maina et al 2017).

In order to develop closed-form solutions for resilient responses of a pavement structure under the rectangular tyre loading, the Cartesian coordinate system may be convenient to use. Butler (1971) derived the theoretical solution for multi-layered systems using the Cartesian coordinate system for isotropic materials, but did not provide any worked examples. Similarly, Ernian (1989) used both the cylindrical and Cartesian coordinate systems to derive solutions for both circular and rectangular uniformly distributed loads acting on the surface of a multi-layered system with isotropic material properties.
This paper presents the development of closed-form solutions for a multi-layered pavement system under static rectangular loadings, considering both isotropic and cross-anisotropic material properties. Isotropic materials have the same elastic properties in both the vertical and horizontal directions, and can be described by three independent elastic constants – elastic modulus \(E\), Poisson’s ratio \(ν\) and shear modulus \(G\). In contrast, cross-anisotropy of an elastic material is defined by five independent elastic constants – two elastic moduli in vertical and horizontal directions \( (E_v, E_h) \), two Poisson’s ratios in vertical and horizontal directions \( (ν_{vh}, ν_{hv}) \) and one shear modulus \( (G_h) \), as presented by Love (1944).

In this research study, two classical mathematical methods, i.e. classical trigonometric integral and classical potential function, were investigated for flexibility and efficiency. The former was adopted in this study and its use in the determination of pavement responses is presented in this paper. This work is an extension of work where solutions of responses due to circular loading were presented (Maina et al. 2017). This method is flexible enough and can easily be extended to dynamic and wave propagation problems, although this is not the focus of the paper.

Accuracy and validity of the solutions are verified through comparisons of the computed responses to results obtained using a proprietary finite element package for a five-layer pavement structure, where the four upper main layers were sub-layered, resulting in a pavement system with 17 layers. The pavement layers were composed of materials with isotropic and cross-anisotropic (also known as transversely isotropic) properties.

**THEORETICAL DEVELOPMENT**

**Three-dimensional linear elasticity**

For 3D problems shown in Figure 1, the differential equilibrium equations in Cartesian coordinates may be expressed using modified Navier’s equations. However, the equations are cumbersome to deal with because of the need to solve three coupled partial differential equations for the three displacement components. The difficulty with finding particular solutions of the system of equations in terms of the displacements arises because each of the sought-after deflection functions in the Cartesian coordinates \((x, y, z)\) appear in all three equilibrium equations.

The solutions may be simplified by representing displacements in terms of harmonic potentials. It is because this approach decouples the equations in various different ways. The most common approach is to use the so-called Papkovich-Neuber potentials to represent the solution (Ozawa et al. 2009, Borodachev & Astanin 2008). This approach enables the use of a well-known catalogue of particular solutions of the Laplace equation, and sometimes even reduces the problem, if not completely, to one of the classical problems of the theory of harmonic functions (theory of potential). Despite the simplification, it is difficult to extend this approach to problems of dynamic or moving load analysis (Ozawa et al. 2010).

This paper aims at presenting closed-form solutions of pavement responses due to static rectangular loading in the vertical direction. The solutions presented in this paper were derived based on a more flexible and efficient classical transform integral method. A similar approach can be followed to derive solutions for rectangular loads acting in the longitudinal and transverse directions.

**Theoretical development**

Three different approaches may be used to solve problems of the theory of elasticity (Borodachev 1995; 2001). In the first approach, the displacement vector is determined first, and this vector is then used to determine the stress and strain tensors (known as problem in displacements).

In the second approach the stress tensor is determined first, and then this tensor is used to determine the strain tensor and displacement vector (known as problem in stresses). In the third approach the strain tensor is determined first, and then stress and displacement tensors are determined (known as problem in strains). The work presented in this paper followed the first approach, namely problem in displacements.

A system of rectangular Cartesian coordinates \((x, y, z)\) is used. By assuming the body forces to be zero, equilibrium equations for an infinitesimal element (Figure 2) can be expressed using Navier’s equations as follows (Filonenko-Borodich 1963):

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (1)
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (2)
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (3)
\]

Where: \(\sigma_x, \sigma_y, \sigma_z\) are normal stress and \(\tau_{xy}, \tau_{xz}, \tau_{yz}\) are shear stresses acting on an infinitesimal element. The two subscripts on the symbols for shear stresses represent, respectively, the face and direction on which the shear stress is acting.
The strain-displacement relationship may be represented as follows:

\[
\begin{align*}
\varepsilon_x &= \frac{\partial u}{\partial x}, \quad \varepsilon_y &= \frac{\partial v}{\partial y}, \quad \varepsilon_z &= \frac{\partial w}{\partial z}, \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.
\end{align*}
\]

(4)

Where: \(u = u(x, y, z)\), \(v = v(x, y, z)\) and \(w = w(x, y, z)\) are displacements in the directions of \(x\), \(y\) and \(z\) axes. Furthermore, \(\varepsilon_x\), \(\varepsilon_y\) and \(\varepsilon_z\) are normal strains corresponding to normal stresses \(\sigma_x\), \(\sigma_y\) and \(\sigma_z\), whereas \(\gamma_{xy}\), \(\gamma_{xz}\) and \(\gamma_{yz}\) are shear strains corresponding to shear stresses \(\tau_{xz}\), \(\tau_{yz}\) and \(\tau_{xy}\).

**Generalised Hooke’s law**

In linear elasticity, if the stress is sufficiently small, Hooke’s law is used to represent the material behaviour and to relate the unknown stresses and strains. The general equation for Hooke’s law is:

\[
\sigma_{ij} = E_{ijkl} \epsilon_{kl} = \sum_{k=1}^{3} \sum_{l=1}^{3} \sigma_{ijkl} \epsilon_{kl}
\]

(5)

where \(i, j = 1, 2, 3\).

In this case \(\sigma_{ijkl}\) is a fourth-rank tensor called elastic compliance of the material. Each subscript of \(\sigma_{ijkl}\) takes on the values from 1 to 3, giving a total of \(3^4 = 81\) independent components in \(s\). However, due to the symmetry of both \(\epsilon_{ij}\) and \(\sigma_{ijkl}\) the elastic compliance \(s\) must satisfy the relation:

\[
s_{ijkl} = s_{jikl} = s_{ijk} = s_{jikl}
\]

(6)

It follows from Equation 5, therefore, that the generalised Hooke’s law in Equation 5 can be simplified to become:

\[
\sigma_{ij} = s_{ijkl} \epsilon_{kl} \quad \text{where } i, j = 1, \ldots, 6
\]

(7)

This relationship reduces the number of \(s\) components to 36, as seen in the following linear relation between the pseudovector forms of the strain and stress:

\[
\begin{align*}
\sigma_1 &= \epsilon_{11}, \\
\sigma_2 &= \epsilon_{22}, \\
\sigma_3 &= \epsilon_{33}, \\
\sigma_4 &= \epsilon_{23}, \\
\sigma_5 &= \epsilon_{32}, \\
\sigma_6 &= \epsilon_{12}.
\end{align*}
\]

(8)

The \(s\) matrix in this form is also symmetric. It therefore contains only 21 independent elements. The number of independent elements is obtained by counting elements in the upper right triangle of the matrix, including the diagonal elements (i.e. \(1 + 2 + 3 + 4 + 5 + 6 = 21\)). Furthermore, if the material exhibits symmetry in its elastic response, the number of independent components in the \(s\) matrix will be reduced even further.

For example, in the simplest case of isotropic materials whose elastic moduli are the same in all directions, only three unique components \(s_{11}, s_{12}, s_{44}\) exist, as shown in Equation 9. In the elastic range, these three coefficients of the ordinary isotropic model require three parameters \((E, v, G)\) for their definitions as follows:

\[
s = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & s_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & s_{44} & 0 & 0 & 0 \\
0 & 0 & 0 & s_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & s_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & s_{44}
\end{bmatrix}
\]

(9)

Where: \(s_{11} = \frac{1}{E} \epsilon_{11} = -\frac{\nu}{E} s_{44} = \frac{1}{G}\)

\[
G = \frac{E}{2(1 + \nu)}
\]

(10)

In cross-anisotropic materials, however, six unique components \((s_{11}, s_{12}, s_{13}, s_{33}, s_{44}, s_{46})\) and \(s_{66}\) exist, as shown in Equation 10. These six components of the ordinary cross-anisotropic model require five parameters \((E_v, E_{hh}, v_{vh}, v_{hv}, G_{ih})\) for their definitions (Love 1944).

\[
s = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & s_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & s_{44} & 0 & 0 & 0 \\
0 & 0 & 0 & s_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & s_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & s_{66}
\end{bmatrix}
\]

(11)

Where:

\[
s_{11} = \frac{1}{E_v} \epsilon_{11} = -\frac{v_{hh}}{E_{hh}} \epsilon_{13} = -\frac{v_{hv}}{G_{ih}} \frac{E_v}{2(1 + \nu_{hv})}, \\
s_{33} = \frac{1}{G_{hh}} \epsilon_{33} = \frac{E_v}{2(1 + \nu_{hv})}, \\
s_{66} = 2(s_{11} - s_{13})
\]

(12)

Writing the stresses in terms of the strains would require Equation 8 to be inverted, yielding:

\[
\epsilon_{ij} = \sigma_{ij} \epsilon_{ij}
\]

(13)

Where: \(\sigma_{ij}\) is a fourth-rank tensor called elastic stiffness of the material. The stiffness and compliance matrices in Equation 11 are related in the following form:

\[
c = s^{-1}
\]

(14)

Three components \((c_{11}, c_{12}, c_{44})\) exist for isotropic materials, whereas, in cross-anisotropic materials, five unique components \((c_{11}, c_{12}, c_{13}, c_{33}, c_{44})\) exist. It should be noted that a special cross-anisotropy solution exists, where \(E_v = E_h\) and \(v_{hv} = v_{hh}\). Equation 13 shows the six components of the elastic stiffness matrix, whereas Equations 14–19 define each of these six components:

\[
c = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}
\]

(15)

Where:

\[
c_{11} = \frac{E_v}{(1 + v_{hh})(E_v + E_{vh} + 2E_{hh}v_{hv})}, \\
c_{12} = \frac{E_hv_{hh}}{(1 + v_{hh})(E_v + E_{vh} + 2E_{hh}v_{hv})}, \\
c_{13} = \frac{E_h}{E_v + E_{vh} + 2E_{hh}v_{hv}}, \\
c_{33} = \frac{E_v^2}{(1 + v_{hh})(E_v + E_{vh} + 2E_{hh}v_{hv})}, \\
c_{44} = \frac{E_v}{2(1 + v_{hh})}, \\
c_{66} = \frac{E_v}{2(1 + v_{hh})}
\]

(16)

Substituting Equation 13 in 11 yields:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix} \begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix}
\]

(17)

**Derivation of the solutions**

It is convenient to use the three-dimensional Cartesian coordinate system and make an assumption that displacement functions \(u(x, y, z)\), \(v(x, y, z)\) and \(w(x, y, z)\) in the \(x\), \(y\) and \(z\) directions, respectively, may be represented using double trigonometric functions, as detailed below. With this approach, the \(x\) and \(y\) dependencies of the displacement functions \(u(x, y, z)\), \(v(x, y, z)\) and \(w(x, y, z)\) are accommodated by means of analytical double integral Fourier transforms, with the \(z\) dependence approximated by using closed-form solutions.

The Fourier transform of the displacements employs transform pairs that are
defined in terms of the Fourier parameters in the x and y directions ($\xi_x$ and $\xi_y$) as follows:

\[
\begin{align*}
\tilde{u}(\xi_x, \xi_y, z) &= \int \int u(z) \sin(\xi_x x) \cos(\xi_y y) \, d\xi_x \, d\xi_y \\
\tilde{v}(\xi_x, \xi_y, z) &= \int \int v(z) \cos(\xi_x x) \sin(\xi_y y) \, d\xi_x \, d\xi_y \\
\tilde{w}(\xi_x, \xi_y, z) &= \int \int w(z) \cos(\xi_x x) \cos(\xi_y y) \, d\xi_x \, d\xi_y
\end{align*}
\] (21, 22, 23)

Where: $\tilde{u}(\xi_x, \xi_y, z)$, $\tilde{v}(\xi_x, \xi_y, z)$ and $\tilde{w}(\xi_x, \xi_y, z)$ are the Fourier transforms of displacement functions about the coordinate z (Sneddon 1951). The procedure that follows is to expand Equation 20 and to express strains in terms of elastic displacements based on Equation 4. After that the Fourier transforms of the displacements using Equations 21, 22 and 23 are applied. Then the resulting functions are substituted into Equations 1–3 to yield their Fourier transforms as follows:

\[
\begin{align*}
\tilde{u}(\xi_x, \xi_y, z) &= (\xi_x \xi_y (c_{11} + c_{12}) + 2c_{13} \xi_x^2) + \\
(\xi_x \xi_y (c_{11} + c_{12}) + 2c_{13} \xi_x^2) \tilde{v}(\xi_x, \xi_y, z) + 2c_{13} (\xi_x + c_{44}) \\
\frac{\partial^2 \tilde{w}(\xi_x, \xi_y, z)}{\partial z^2} - 2c_{44} \frac{\partial^2 \tilde{u}(\xi_x, \xi_y, z)}{\partial z^2} &= 0
\end{align*}
\] (24)

Using Equations 27 and 28 it becomes convenient to eliminate $\partial(\xi_x, \xi_y, z)$ and $\partial(\xi_x, \xi_y, z)$ from Equations 24–26, then simplify to obtain:

\[
(\lambda^4 - \lambda_1 \lambda^2 + \lambda_2 \lambda + \lambda_3)\tilde{w} = 0
\] (29)

The roots of Equation 29 are determined as:

\[
\lambda = \pm \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}}
\] (30)

Where: $\lambda_1 = (b + (a - d) - c f)$, $\lambda_2 = a + b + d$ from Equation 27.

Putting back the differential function in Equation 29 yields:

\[
\frac{d^4}{dz^4} - \left(\frac{b}{a} \frac{d^2}{dz^2} + \frac{c}{a} \frac{d}{dz}\right)\tilde{w} = 0
\] (31)

With reference to the roots in Equation 30, the solution form of Equation 31 depends on the sign of the coefficient $\lambda_2 - 4\lambda_2$.

Furthermore, and this is very important, depending on the numerical integration methods used, it may be necessary to modify the solution for $\tilde{w}$ to obtain stable and accurate solutions of the pavement structure. The solutions presented hereunder were based on the numerical methods used in this research.

1. Solution 1: $\lambda_1^2 - 4\lambda_2 > 0$, where $\lambda_1 > \lambda_2$

\[
\tilde{w} = \tilde{w}(\xi_x, \xi_y, z) = C_1(\xi_x)^{\xi_1} + C_2(\xi_y)^{\xi_2} + C_3(\xi_x)^{\xi_3} + C_4(\xi_y)^{\xi_4}
\] (32)

Where:

\[
\begin{align*}
\lambda_1 &= \xi_1 \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}} \\
\lambda_2 &= \xi_2 \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}}
\end{align*}
\]

and $C_1(\xi_x), C_2(\xi_y), C_3(\xi_x)$, and $C_4(\xi_y)$ are coefficients of integration determined, as described later, by using boundary loading conditions.

In order to obtain stable and accurate results of pavement responses, Equation 32 was modified to:

\[
\tilde{w}(\xi_x, \xi_y, z) = C_1(\xi_x) \cosh(r z) e^{i\xi z} + C_2(\xi_y) \cosh(r z) e^{-i\xi z} - C_3(\xi_x) \sinh(r z) e^{i\xi z} - C_4(\xi_y) \sinh(r z) e^{-i\xi z}
\] (33)

Where:

\[
\begin{align*}
\kappa_1 &= \xi_1 \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}} \\
\kappa_2 &= \xi_2 \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}}
\end{align*}
\]

and $C_1(\xi_x), C_2(\xi_y), C_3(\xi_x)$, and $C_4(\xi_y)$ are coefficients of integration determined, as described later, by using boundary loading conditions.

2. Solution 2: $\lambda_1^2 - 4\lambda_2 = 0$, where $\lambda_1 = \lambda_2$

\[
\tilde{w}(\xi_x, \xi_y, z) = e^{\xi x} \left( C_1(\xi_x) \xi_x + C_2(\xi_y) \xi_y \right)
\] (34)

3. Solution 3: $\lambda_1^2 - 4\lambda_2 < 0$, where $\lambda_1 < \lambda_2$

\[
\tilde{w}(\xi_x, \xi_y, z) = C_1(\xi_x)^{\xi_1} + C_2(\xi_x)^{\xi_2} + C_3(\xi_y)^{\xi_3} + C_4(\xi_y)^{\xi_4}
\] (35)

For stable and accurate results of pavement responses, Equation 35 was modified to:

\[
\tilde{w}(\xi_x, \xi_y, z) = C_1(\xi_x) \cos(r z) e^{i\xi z} + C_2(\xi_y) \cos(r z) e^{-i\xi z} - C_3(\xi_x) \sin(r z) e^{i\xi z} - C_4(\xi_y) \sin(r z) e^{-i\xi z}
\] (36)

\[
\begin{align*}
\lambda_1 &= \xi_1 \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}} \\
\lambda_2 &= \xi_2 \sqrt{\frac{\lambda_1 + \sqrt{\lambda_1^2 - 4\lambda_2}}{2}}
\end{align*}
\]

Solutions for the remaining Fourier transformed displacements

Substituting Equations 33, 34 and 36 into Equations 24 and 25, the solutions for $\tilde{u}(\xi_x, \xi_y, z)$ and $\tilde{v}(\xi_x, \xi_y, z)$ are derived.

As an example, solutions for $\tilde{u}(\xi_x, \xi_y, z)$ and $\tilde{v}(\xi_x, \xi_y, z)$ for the case where $\lambda_1 = \lambda_2$ (Solution 2) are obtained as follows:

\[
\tilde{u}(\xi_x, \xi_y, z) = C_1(\xi_x)^{\xi_1} + C_2(\xi_y)^{\xi_2} + C_3(\xi_x)^{\xi_3} + C_4(\xi_y)^{\xi_4}
\] (37)

\[
\tilde{v}(\xi_x, \xi_y, z) = e^{i\xi x} \left( C_1(\xi_x) \xi_x + C_2(\xi_y) \xi_y \right)
\] (38)

Solutions for $\tilde{u}(\xi_x, \xi_y, z)$ and $\tilde{v}(\xi_x, \xi_y, z)$ are then substituted into Equations 4 and 20 to determine the Fourier transforms of normal and shear.
For the loading case shown in Figure 3, there is only a single uniformly distributed surface rectangular vertical load, \( P \), whose sides are \( 2 \times a \) and \( 2 \times b \) in dimensions. Boundary conditions for rectangular loads acting on a surface of a semi-infinite medium shown in Figure 3 may be represented by taking into consideration the equilibrium between external and internal vertical and shear stresses, as shown below:

When \( x \leq |a | \) and \( y \leq |b| \) then:

\[
\begin{align*}
\sigma_x^1(x, y, z) &= p \cos(\xi_x y) \\
\sigma_y^1(x, y, z) &= 0 \\
\tau_x^1(x, y, z) &= 0
\end{align*}
\]

Where:

\[
p_1^1(x, y, z) = \left\{ \begin{array}{ll}
p \cos(\xi_x y) \\
0
\end{array} \right. \quad \xi_x y \leq \frac{\pi}{2} - a \\
0 \quad \xi_x y > \frac{\pi}{2} - a
\]

And:

\[
p_2^1(x, y, z) = \left\{ \begin{array}{ll}
p \cos(\xi_y x) \\
0
\end{array} \right. \quad \xi_y x \leq \frac{\pi}{2} - b \\
0 \quad \xi_y x > \frac{\pi}{2} - b
\]

Boundary condition – surface rectangular vertical loading

For the loading case shown in Figure 3, there is only a single uniformly distributed surface rectangular vertical load, \( P \), whose sides are \( 2 \times a \) and \( 2 \times b \) in dimensions. Boundary conditions for rectangular loads acting on a surface of a semi-infinite medium shown in Figure 3 may be represented by taking into consideration the equilibrium between external and internal vertical and shear stresses, as shown below:

When \( x \leq |a | \) and \( y \leq |b| \) then:

\[
\begin{align*}
\sigma_x^1(x, y, z) &= p \cos(\xi_x y) \\
\sigma_y^1(x, y, z) &= 0 \\
\tau_x^1(x, y, z) &= 0
\end{align*}
\]

Where:

\[
p_1^1(x, y, z) = \left\{ \begin{array}{ll}
p \cos(\xi_x y) \\
0
\end{array} \right. \quad \xi_x y \leq \frac{\pi}{2} - a \\
0 \quad \xi_x y > \frac{\pi}{2} - a
\]

And:

\[
p_2^1(x, y, z) = \left\{ \begin{array}{ll}
p \cos(\xi_y x) \\
0
\end{array} \right. \quad \xi_y x \leq \frac{\pi}{2} - b \\
0 \quad \xi_y x > \frac{\pi}{2} - b
\]

In addition, when \( x > |a | \) and \( y > |b| \) then:

\[
\begin{align*}
\sigma_x^1(x, y, z) &= 0 \\
\sigma_y^1(x, y, z) &= 0 \\
\tau_x^1(x, y, z) &= 0
\end{align*}
\]

Pavement responses

By applying Fourier inverse transformation of all the Fourier transformed solutions, it is possible to determine solutions for pavement responses at any point in a pavement structure.

\[
u(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{u}(k_x, k_y, z) \cos(\xi_x x) \cos(\xi_y y) \; dk_x dk_y
\]

\[
v(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{v}(k_x, k_y, z) \cos(\xi_x x) \sin(\xi_y y) \; dk_x dk_y
\]
This study

The pavement structure for which the simulation
results are presented here is shown in Figure 4. Based on historical tyre loading on South African roads, a tyre vertical load of 21.5 kN on a rectangular patch of 231 mm by 238 mm resulting in a 390 kPa contact stress was used in the analysis (De Beer 2008). All layers, except the asphalt layer, were modelled with isotropic, linear-elastic properties. The asphalt layer was modelled with cross-anisotropic, linear elastic properties. Information on all the layers, including sub-layers, is shown in Table 1. The sub-laying of the upper four main layers into four layers each resulted in a total of 17 layers, as shown in Table 1.

The vertical stiffness in the asphalt sub-layers shows a marginal increase close to the surface that may be attributed to binder ageing, but more probably to a slight reduction in the temperature conditions. The top and bottom asphalt sub-layers show significant reduction in the effective horizontal stiffness resulting from cracking initiating from both the top and bottom of the main asphalt layer.

Discussion of results

Figures 5 and 6 show comparisons between the numerical method developed in this study and a proprietary FEM package for stresses (\(S_{zz}\)) and (\(S_{xx}\)) as well as strains (\(E_{zz}\)) and (\(E_{xx}\)) distributions with depth, z-direction, under the centre of a rectangular surface load of magnitude 21.5 kN (390 kPa contact stress). From these results it is clear that the closed-form solutions developed in this study have achieved a very good level of accuracy, as their results compare well with results from a proprietary FEM package.

It is also important to mention here that the software developed from this work has become the analysis engine for the new SAPDM (South African Pavement Design Method).

What is also evident in the strains plots is that, as the FEM increases the size of the elements at points far from where the load is acting, the accuracy is reduced a little bit and the results start moving away from the results of this study. However, all in all, the agreement of the two methods, i.e. approximation by FEM and closed-form solution developed in this paper, is very good.

CONCLUSIONS

1. A numerical tool for analysis of an elastic multilayer system under the action of a surface rectangular load, considering both cross-anisotropic and isotropic material properties, has successfully been developed.
2. The numerical tool developed in this study is capable of performing analyses for an unlimited number of points in an elastic multi-layered pavement system with an unlimited number of layers, and on the surface where an unlimited number of uniformly distributed rectangular loads act.

WORKED EXAMPLES

Solutions developed in this study were used to compute responses at different positions within a pavement structure. The pavement structure for which the simulation

\[
w(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{w}(\xi_x, \xi_y, z) \cos(\xi_x x) \sin(\xi_y y) d\xi_x d\xi_y
\]

\[
\sigma_x(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\sigma}_x(\xi_x, \xi_y, z) \cos(\xi_x x) \cos(\xi_y y) d\xi_x d\xi_y
\]

\[
\sigma_y(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\sigma}_y(\xi_x, \xi_y, z) \cos(\xi_x x) \sin(\xi_y y) d\xi_x d\xi_y
\]

\[
\tau_{xz}(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\tau}_{xz}(\xi_x, \xi_y, z) \cos(\xi_x x) \sin(\xi_y y) d\xi_x d\xi_y
\]

\[
\tau_{xy}(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\tau}_{xy}(\xi_x, \xi_y, z) \sin(\xi_x x) \sin(\xi_y y) d\xi_x d\xi_y
\]
3. The results shown in this paper confirm the accuracy and reliability of the closed-form theoretical solutions developed.

4. A very good match of the stress results was obtained between the numerical tool developed in this study and a proprietary FEM package.

5. Differences in the strain results at points far from where the load acts, were observed. The differences seem to be emanating from the FEM, which is a proprietary package. In conventional FEM, stress computations come after strain computations, and the trends should have been similar. The reason for the differences is not clear, but the results, as measured, are nevertheless reported here.

6. The numerical tools developed can be used to improve the design, evaluation and analysis of multi-layered road/runway pavement systems.

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REFERENCES


Potential solution to pollution of groundwater by diffusion of volatile organic compounds through the primary HDPE geomembrane in composite lining systems of landfills

R A Pienaar, W Fourie

Waste in a landfill is exposed to the chemicals and heat generated over time, producing harmful fluids in the form of leachate or landfill gas that migrate from the landfill towards the liner or capping, and include organic contaminants. The high-density polyethylene (HDPE) geomembrane (GM) component of the landfill liner is often believed to be the primary barrier to contaminant transport, but volatile organic compounds (VOCs) diffuse through geomembranes at appreciable rates. The aim of this study was to obtain reliable data on the reduction in diffusion of VOCs through the HDPE GM component in the composite liner systems of landfills by extracting air through the leakage detection layer or drainage layer of the composite liner. It was shown that introducing a flow of air through a pervious zone adjacent to the GM layer in a landfill liner would significantly reduce the concentrations of VOCs in the groundwater beneath landfills and waste containment facilities.

INTRODUCTION AND BACKGROUND

Waste and the adverse effects of waste storage and disposal is not a modern concept. It has always been a byproduct of human beings’ use of the earth’s natural resources for survival.

The safe and reliable long-term disposal of solid waste residues is an important component of integrated waste management. Solid waste residues are waste components that are not recycled, that remain after processing at a material recovery facility, or that remain after the recovery of energy. Historically, solid waste was placed in depressions in the soil of the earth’s crust through a process called landfiling.

When waste materials in a landfill or any other waste body are exposed to the air and water infiltration over time, heat is generated and harmful fluids in the form of leachate and/or landfill gas are generated. Leachate and landfill gas migrate from the landfill towards the basal liner or capping, and most often include organic contaminants. These organic contaminants include a group commonly referred to as volatile organic compounds (VOCs) that have been known to migrate to and pollute the underlying groundwater (Prosser & Janechek 1995).

The high-density polyethylene (HDPE) geomembrane (GM) as part of a landfill liner is often believed to be the primary barrier to contaminant transport, but the clay component in the composite liner usually controls the rate of transport of VOCs, since researchers like Edil (2003) have shown that VOCs diffuse through geomembranes at appreciable rates. Therefore, the effectiveness of modern landfill liner systems in minimizing the migration of VOCs merits scrutiny.

Landfill and landfill liner technology has gone through significant developments in recent years. Waste disposal landfills (general and hazardous) have evolved from controlled dumps to highly engineered facilities designed to protect the environment and promote environmental sustainability. Liner technology and the relevant regulations that govern them have also evolved from rudimentary compacted clay liners to complex composite engineered lining systems comprising a range of layers such as compacted clay, geosynthetic clay liners, geomembranes, geocomposite drains and geotextiles.

Early concerns regarding composite liners typically focused on their hydraulic conductivity and their ability to limit advective transport (Edil 2003), but evidence has been presented subsequently that highlights diffusive transport (i.e. contaminant migration driven by the difference in concentration between the upper and lower sides of the liner) as a
dominant mode of transport in well-built liner systems (McWatters & Rowe 2009).

Although HDPE GMs are used for a variety of applications as barriers for contaminant transport, for the purpose of this study, the only application of HDPE investigated will be that of waste disposal landfill liner.

**STUDY OBJECTIVES**
The aim of this study was to obtain reliable data on the reduction in diffusion of VOCs through the HDPE GM component in composite liner systems of landfills by extracting air through the leakage detection layer or drainage layer of the composite liner adjacent to the HDPE component of the liner. With the extraction of air through the liner system, some of the VOCs that could diffuse through the HDPE GM layer would be extracted with the air and thus reduce VOCs in the underlying soil and/or groundwater.

The objective was to undertake tests in the following three phases:
- **Phase 1** aimed to prove that the chosen VOCs diffuse from source to receptor through an HDPE GM layer, and to compare the results obtained with those in the literature.
- **Phase 2** aimed to prove that, even if the separation between the source and receptor consisted of two HDPE GMs separated by an air-filled pervious zone, diffusion of the VOCs would still occur from the source to the receptor volumes.
- **Phase 3** aimed to prove that, by introducing airflow into the pervious zone between the two HDPE GMs, the concentration of VOCs in the receptor volume (due to diffusion through the HDPE GM) could be reduced significantly. The testing in this phase also aimed to determine if the rate of air removal would play a role in the diffusive process and the resultant VOC concentrations in the receptor.

For the purposes of this study, the onsite conditions were replicated in a laboratory using diffusion test cells similar to those used by Sangam and Rowe (2001). All of the HDPE GMs used in this study were supplied by Aquatan (Pty) Ltd in South Africa, and the diffusion test cells were built by Interlock Systems (Pty) Ltd in Pretoria, South Africa.

**LITERATURE REVIEW**

**Waste disposal and containment barriers**
In South Africa, waste disposal landfills are grouped into four classes according to the waste types earmarked for disposal (Classes A, B, C and D).

**Figure 1** Class A containment barrier design prescription (Government Gazette 36784 p 37)

Class A landfills require a minimum of a double-composite containment barrier system and are meant for the disposal of hazardous wastes. The Class A landfill liner prescribed in the Waste Classifications and Management Regulations of the Waste Act (DWAF 1998) are presented in Figure 1.

The liner detail in Figure 1 is a general minimum standard, and every containment facility needs to have its own fit-for-purpose engineered lining system that conforms to the Class of facility and waste type for which it is intended. The layers can be replaced by other layers of equal or improved performance, and the compacted clay layers are often replaced with a geosynthetic clay liner (GCL). The leakage detection system normally made up of granular material can also be replaced by an approved geosynthetic equivalent alternative such as a cusped HDPE drainage sheet or geocomposite drain. A fluid could pass through this layer in order to remove VOCs from the system and possibly cool the liner. HPDE GMs have been widely used in landfill and waste containment barriers due to their high resistance to advective flow of leachate and resistance to chemical attack (Islam and Rowe 2009).

Although the focus of this study was the HDPE GM, in most cases the HDPE GM is used as a primary barrier in conjunction with other engineered layers to form a composite lining system that is designed and engineered to be fit for purpose. The HDPE GM is an integral part of a composite landfill lining system, and in South Africa the Department of Environmental Affairs has included the HDPE GM in the liner requirements for the successful application of any waste licence to own or operate a waste disposal facility.

HDPE GMs remain fit for purpose in landfill liner applications for up to 1 000 years (Rowe 2005), depending on a range of factors, such as the period of exposure to active leachate, the height of waste on the liner, the chemical composition of the waste being contained and the temperature of the waste body. The temperature that the HDPE GM is exposed to has a significant impact on the service life of the HDPE and Rowe (2005) has shown that HDPE GM service life (or half-life), based on 50% reduction in tensile strength at break, can be between 565 and 900 years when exposed to temperatures not exceeding 20°C, but can reduce to as little as 15–20 years when exposed to temperatures of more than 60°C.

Thus, for an HDPE GM to be a successful barrier in the landfill liner, it needs to be manufactured, installed and monitored according to the specifications given by design engineers and manufacturers.

**Volatile organic compounds**
Volatile organic compounds (VOCs) are organic chemical compounds that have high enough vapour pressures under normal
Diffusion is often referred to as molecular diffusion of VOCs in the environment accidentally, where they can characterize organic compounds and to mixtures of variable composition. Sometimes VOCs are released into the environment accidentally, where they can contaminate soil and groundwater, for example the deposition of waste and waste-related products into engineered waste disposal landfills equipped with an engineered lining system. VOCs have been shown to diffuse through the lining systems of landfills, resulting in, among other things, groundwater pollution (Touze-Foltz et al. 2011).

Methane is the most commonly known VOC and, as a greenhouse gas, is a significant contributor to environmental pollution. Methane generally migrates to the surface of landfills, while other VOCs, such as the aromatic hydrocarbons (benzene, toluene, ethylbenzene, xylene (BTEX)) commonly found in petroleum products, migrate to the base of the landfill and contribute to groundwater contamination (Prosser et al. 2011).

In recent years, the BTEX hydrocarbons have attracted much attention because they constitute one of the most common and serious threats to groundwater reservoirs in close proximity to contaminated sites. The VOCs used for this study are BTEX (the xylene being p-xylene) and chloroform. These contaminants were selected because they are commonly found in landfill leachates (Rowe 2005) and are also significant contributors to groundwater contamination. Exposure to VOCs in the short and long term can be detrimental to human health in various ways.

**Diffusion of VOCs**

Diffusion is often referred to as molecular diffusion and is the net transport of molecules from a region of higher concentration to one of lower concentration by random molecular motion. Due to the nature of the material and how it is manufactured, intact HDPE GMS prevent advective flow of contaminants through their structure, hence their widespread use in the containment of water and other liquids. HDPE GMs do, however, allow movement of certain contaminants through their structure by means of molecular diffusion (Rowe 1998).

The diffusive movement of contaminants through an intact GM with no faults or holes involves a cooperative rearrangement of the penetrant molecule and the surrounding polymer chain segments. For the penetrant molecule to move into the polymer structure of the HDPE GM, the process requires the localization of energy to be available (Rowe 1998). Thus, the diffusive motion requires energy and depends on the relative mobility of the penetrant molecules in the contaminant or leachate and polymer chains in the HDPE. In turn, this will depend on temperature, concentration, the size and shape of the penetrant and the nature of the polymer itself (Rowe 1998).

Sangam and Rowe (2001) describe the molecular diffusion of penetrants such as BTEX and chloroform through an intact HDPE GM as a molecular-activated process that occurs in a series of steps following the path of least resistance. For diluted aqueous solutions, as is the case in this study, this involves three steps (Park & Nibras 1993):

1. **Adsorption** (partitioning of contaminant between the inner surface of the HDPE GM and the medium containing the contaminant).
2. **Diffusion** of the permeant through the HDPE GM.
3. **Desorption** (partitioning of the contaminant between the outer surface of the HDPE GM and the outer medium).

For water or water-based solutions like those used in this experiment, the adsorption and desorption processes can be seen as similar and inverted (Sangam & Rowe 2001). In the Keynote Lecture for the 6th International Conference on Geosynthetics held in Atlanta in 1998, Professor Kenny Rowe presented a schematic drawing similar to the one shown in Figure 2 to illustrate the diffusive transport of contaminants through an HDPE GM. The figure shows partitioning between the concentration in solution and the concentration dissolved in the GM.

Figure 2 illustrates that the process starts with the removal of the molecule from the solution fluid onto the surface of the HDPE GM (Step 1: Adsorption). The sorption of the permeant onto the polymer (the HDPE GM) depends on a variety of factors and thus the extent to which permeant molecules are sorbed, and their mode of sorption in a polymer depends upon the activity of the permeant within the polymer at equilibrium (Mueller et al. 1998). For the simplest case where the permeant does not interact with the polymer (as is the case for a HDPE GM) or at low concentrations (as in landfill leachates), the relationship between the concentration in the fluid and the HDPE GM (solid) is given by the Nernst distribution function which takes the linear form shown in Equation 1 (Henry’s Law) (Sangam & Rowe 2001):

\[
C_z = S_{gf} C_f
\]  

(1)

Where: \(S_{gf}\) is called a partitioning or sorption coefficient [-] and, in principle, is a constant for the given molecule, fluid, HDPE GM and temperature of interest, and \(C_z\) and \(C_f\) are the concentrations of the permeant in the HDPE GM and the fluid respectively.

For organic compounds in aqueous solution, like BTEX and/or chloroform in water, the value of \(S_{gf}\) is strongly related to the solubility of the compound of interest in water (Rowe 1998). The lower the solubility in water, the higher the affinity of the HDPE GM to attract the compound, and thus the higher \(S_{gf}\) will be when in aqueous solution (Rowe 1998). Compounds with high solubility thus generally give lower \(S_{gf}\) values.

The process ends with desorption (Step 3: Desorption), which is similar to adsorption and, for an aqueous solution in contact with a HDPE GM, it can be assumed that Equation 1 also holds true, meaning the partitioning coefficient for adsorption and desorption of BTEX and chloroform in aqueous solutions are equal (Sangam & Rowe 2001).

The diffusion process in the HDPE GM happens between the adsorption and desorption processes and can be explained by Fick’s First Law:

\[
f = -D_z \frac{\partial C_z}{\partial z}
\]  

(2)

Where: \(f\) is the rate of transfer per unit area [ML⁻²T⁻¹] (typically mg per m² per second), \(D_z\) is the diffusion coefficient in the HDPE GM [L²T⁻¹] (typically m² per second), \(C_z\) is the concentration of the substance that is diffusing, and \(z\) is the direction parallel to the direction of the diffusion (typically the thickness of the HDPE GM).

\[
\frac{\partial C_z}{\partial t} = D_z \frac{\partial^2 C_z}{\partial z^2}
\]  

(3)
This equation needs to be solved for the appropriate boundary and initial conditions to obtain the diffusion coefficient of the solution / HDPE GM system at equilibrium.

To measure the concentration change in the HDPE GM when doing diffusion tests is difficult, so it is useful to express the diffusion equations in terms of the concentration in adjacent solutions (Sangam & Rowe 2001). Equation 1 gives the relationship between the concentrations in the GM and the adjacent fluid. Equation 3 gives the flux (diffusion) within the HDPE GM, so substituting Equation 1 into Equation 2 gives the flux on one side of the HDPE GM to a similar fluid on the other side of a HDPE GM (Rowe 1998), i.e. Equation 4.

\[
f = -D_g \frac{\partial c_f}{\partial z} = -S_{gf} D_g \frac{\partial c_f}{\partial z} = -P_g \frac{\partial c_f}{\partial z}
\]

Where: \(P_g = S_{gf} D_g\) \(\text{(5)}\)

\(P_g\) gives the relationship between the diffusion coefficient and the sorption coefficient, and is referred to in polymer literature as the permeability coefficient (Sangam & Rowe 2001). This permeability coefficient \(P_g\) should not be confused with the soil mechanics term coefficient of permeability, which more often is called the hydraulic conductivity, or the intrinsic permeability of a porous medium. It has nothing to do with Darcy’s Law or the flow through the open voids within porous media, but accounts for the effects of both diffusion and partitioning.

Based on Equation 5, the mass flux across an HDPE GM of thickness \(t_{GM}\) is thus given by:

\[
f = S_{gf} D_g \frac{\Delta c_f}{t_{GM}} \text{ \(\text{(6)}\)}
\]

Where: \(S_{gf} D_g\) can be replaced by \(P_g\) (Equation 5) and where \(\Delta c_f\) is the difference in concentration in the fluid on either side of the GM (\(c_f\) and \(c_{f0}\) in Figure 2).

The purpose is thus to determine the \(S_{gf}\) and \(D_g\) (and thus \(P_g\)) values of the system in question, and compare them to the values found in the literature, before trying to prove that VOCs can be extracted successfully through a pervious zone in the liner system.

For the purpose of this study, it was the intention to draw air through a pervious zone in the liner system, thereby removing VOCs that would have contaminated the groundwater. This is simulated by separating the source and receptor volumes using two HDPE GMS with an air void between them. The two GMS would be identical and the theory applied for the calculation of the sorption and diffusion coefficients will remain. A study by McWatters and Rowe (2009) investigated the transport of VOCs through GMS from both aqueous and vapour phases and found that: “… diffusive transport of VOC contaminants through geomembranes in a simulated landfill environment is identical despite the phase they originated from, simplifying the analysis of contaminant transport”.

This principle would be adopted in the work for this dissertation study, indicating that diffusive transport should still occur across a system similar to the description above.

**Calculating coefficients in diffusion process**

When undertaking diffusion tests the sorption \(S_{gf}\) and diffusion \(D_g\) coefficients need to be calculated in order to understand and comment on the diffusion process.

\[
S_{gf} = \frac{C_{fg} V_s - C_{gf} (V_s + V_r) - \Sigma V_i C_i}{A t_{GM} C_{jF}} \text{ \(\text{(7)}\)}
\]

Where:

- \(C_{fg}\) is the initial concentration of fluid in the source reservoir [ML\(^{-3}\)]
- \(V_s, V_r\) are the volumes of the source and receptor reservoirs [L\(^3\)]
- \(C_{jF}\) is the final equilibrium concentration in the source and receptor reservoirs [ML\(^{-3}\)]
- \(\Sigma V_i C_i\) is the mass removed by sampling events [M] (\(V_i\) and \(C_i\) being the volume and concentration removed at each sampling event)
- \(A\) is the area of the GM through which diffusion occurs [L\(^2\)]
- \(t_{GM}\) is the thickness of the GM [L].

The diffusion coefficient \(D_g\) is then inferred by using Equation 3 and the variation in source and receptor concentrations with time (Fick’s Second Law) at the given boundary conditions. This is done using POLLUTE v7\(^{®}\), which solves the one-dimensional contaminant migration equation subject to boundary conditions at the top and bottom of the GM being modelled (Sangam & Rowe 2005).

**EXPERIMENTAL METHODOLOGY**

Laboratory tests were carried out at the University of Pretoria in South Africa. The tests undertaken were based on the methods used by Sangam and Rowe (2001).

**Sorption tests**

Sorption tests were done to determine the sorption coefficient \(S_{gf}\) for the HDPE GM and permeant in question. The coefficient is defined as the ratio of the concentration of the chemical in the HDPE GM at equilibrium to the concentration of the chemical in the solution in contact with the HDPE GM (Park & Nibras 1993). \(S_{gf}\) is most often unitless and, when doing diffusion tests where the concentration of contaminants in the source and receptor is monitored over time, can be calculated using Equation 7, and can then be used to infer the diffusion coefficient \(D_g\) using the computer software program POLLUTE\(^{®}\) which was first developed by R K Rowe and J R Booker in 2004 (POLLUTE 2004). This program implements a one-and-a-half dimensional solution to the advection-dispersion equation (Equation 3).

The diffusion coefficient \(D_g\), or so-called diffusivity, has the dimensions of [length\(^2\) time\(^{-1}\)], which result from the underlying kinetic theory. It is a proportional constant between the molar flux due to molecular diffusion and the gradient in the concentration of the species (or the driving force for diffusion). Generally, it is prescribed for a given pair of species, but for a multi-component system, it is prescribed for each pair of species in the system.

The higher the diffusivity (of one substance with respect to another), the faster they diffuse into each other; thus the higher the diffusion coefficient of the VOC in question, given a certain GM and concentration profile, the faster diffusion will occur through the GM into the underlying groundwater.

When the flux across one HDPE GM is investigated, the \(S_{gf}\) value can be determined in various ways, as described by Rowe (1998), and Sangam and Rowe (2001). For VOCs, however, the best method is the diffusion test method, which is the diffusion from solution on one side of the GM to solution on the other side, and monitoring the change in concentration in the source and receptor over time until equilibrium is reached (no significant change in the concentrations in the source and receptor volumes). The value for \(S_{gf}\) is then calculated using Equation 7 (Rowe 1998).

**Sorption tests**

Sorption tests were conducted to determine the sorption coefficient \(S_{gf}\) for the HDPE GM and permeant in question. The coefficient is defined as the ratio of the concentration of the chemical in the HDPE GM at equilibrium to the concentration of the chemical in the solution in contact with the HDPE GM (Park & Nibras 1993). Sorption tests were done using glass vials with sampling caps, about 80 mm high and 50 mm diameter, as shown in Figure 3. The glass sorption cells are similar in shape and size to the glass sorption cells used by Sangam and Rowe (2001).
Diffusion tests were done following the examples given in the work by McWatters and Rowe (2009) to determine the rate of diffusion of the VOCs through the HDPE GM by measuring the change in concentrations of solutions on either side of the HDPE GM. Diffusion tests were carried out in three phases.

**Phase 1 – Diffusion tests using one HDPE GM:** This test would replicate work already done by others in order to prove that, by using the equipment and laboratory setup for this project, diffusion would take place across a 2 mm intact HDPE GM separating a source and receptor volume (see Figure 2) of a diffusion test cell.

**Phase 2 – Diffusion tests using two HDPE GMs:** The second phase of testing replicated the first, with the 2 mm HDPE GM being replaced by 2 x 1 mm HDPE GMs separated by an 8 mm gap filled with air to replicate the top and bottom of an HDPE cusped leakage detection system. The purpose was to prove that diffusion would take place across both HDPE GMs (separated by air in the pervious zone) and still reach groundwater beneath the liner system. The two HDPE GMs were assumed identical since they were cut from the same roll and when there is no flow through the system the VOC concentration in the water above and below the HDPE GMs will reach equilibrium with the concentration of the VOC in the air layer, resulting in the $S_{gf}$ and $D_g$ values being the same for the two HDPE GMs.

**Phase 3 – Extraction of air:** The third phase replicated Phase 2, with air being extracted through the gap between the two HDPE GMs to represent the flow of a fluid through the leakage detection system of a landfill liner. The purpose was to prove that, by removing the air between the two HDPE GMs at regular intervals, the VOCs would be removed from the system and would not reach the groundwater. The two HDPE GMs were assumed identical, but since there would now be a flow of air through the gap between the two GMs, the concentration profile would change, resulting in a change of flux, which could result in a change in the $D_g$ values of the two HDPE GMs.

Stainless steel, which has been used by several investigators to examine the diffusion of BTEX compounds (Sangam & Rowe 2001) was used to manufacture the diffusion cells, and the cells were designed to replicate the diffusion test cells used by Professor Rowe at the Queens University in Kingston, Canada. The test cells were made in South Africa by Interlock Systems and had the dimensions and properties shown in Figure 4.

Five cells were made so that tests could be done in triplicate (for each phase of testing), with one control cell and one blank cell to measure losses and outside influences. The receptor reservoir represented the groundwater beneath lined landfill facilities and was filled with deionised water at the start of testing. The source reservoir represented the leachate in a lined landfill and was filled with a prepared synthetic leachate solution using the filling port.

For the Phase 1 diffusion tests, the source and receptor cells were separated only by the HDPE GM, and during Phases 2 and 3 a centre-piece was added to introduce the pervious zone into the liner system. The centre-piece was separated from the source and receptor reservoirs by HDPE GMs so that the configuration was Source-GM-Pervious Zone-GM-Receptor. During Phase 3 testing the holes in the centre-piece were used to introduce air flow to the system. Using four test cells (A to D), different air flow rates were introduced in three of the cells, leaving one cell permanently closed as for Phase 2 testing in order to provide a control. The air flows were achieved by replacing the air in the pervious zone between the GMs with clean ambient air once every 24 hours, 72 hours and 7 days. In addition to the sorption test described earlier, the $S_{gf}$ value was also obtained from the diffusion test using Equation 7 after the completion of diffusion tests at equilibrium. The diffusion coefficient ($D_g$) was then inferred using Equation 3, and the variation in source and receptor concentrations with time (Fick’s Second Law) at the given boundary conditions using the software POLLUTE®, which solves the one-dimensional contaminant migration equation subject to boundary conditions at the top and bottom of the GM being modelled (Sangam & Rowe 2005).

### RESULTS

**Sorption**

Table 1 summarises the averaged and corrected $S_{gf}$ values obtained.

<table>
<thead>
<tr>
<th>VOC</th>
<th>Aqua sorption $S_{gf}$</th>
<th>Diffusion test $S_{gf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>27.8</td>
<td>14.1</td>
</tr>
<tr>
<td>Toluene</td>
<td>61.9</td>
<td>198</td>
</tr>
<tr>
<td>Ethylbenzene</td>
<td>87.2</td>
<td>326</td>
</tr>
<tr>
<td>p-Xylene</td>
<td>80.2</td>
<td>102</td>
</tr>
<tr>
<td>Chloroform</td>
<td>25.1</td>
<td>14.2</td>
</tr>
</tbody>
</table>

**Diffusion**

**Phase 1 – Diffusion tests using one HDPE GM**

Concentrations in the source and receptor cells were measured, averaged and plotted
The concentration versus time output graphs that POLLUTE® produces if the methodologies described earlier were correctly followed, were combined with the actual laboratory test results and are shown in Figures 7 to 10 for Phase 1 testing.

The Phase 1 diffusion coefficients obtained are then given in Table 2 (chloroform was not tested during Phase 1).

### Phase 2 – Diffusion tests using two HDPE GMs
Concentrations in the source and receptor cells were measured, averaged and plotted (initial concentration over measured concentration) against time for the source Figure 5 and receptor Figure 6 volumes.

The Phase 1 diffusion coefficients for Phase 1 testing

<table>
<thead>
<tr>
<th>VOC</th>
<th>Diffusion coefficient in m²/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>$9.26 \times 10^{-13}$</td>
</tr>
<tr>
<td>Toluene</td>
<td>$8.68 \times 10^{-13}$</td>
</tr>
<tr>
<td>Ethylbenzene</td>
<td>$1.39 \times 10^{-12}$</td>
</tr>
<tr>
<td>p-Xylene</td>
<td>$2.32 \times 10^{-12}$</td>
</tr>
</tbody>
</table>
concentration) against time for the source Figure 11 and receptor Figure 12 volumes.

As for Phase 1 testing, the diffusion coefficient $D_g$ was determined using POLLUTE®. The concentration versus time output graphs that POLLUTE® produced were combined with the actual laboratory test results and then plotted as shown in the example for benzene in Figure 13.

The Phase 2 diffusion coefficients obtained are then given in Table 3.

### Table 3 Calculated diffusion coefficients ($D_g$) for Phase 2 testing

<table>
<thead>
<tr>
<th>VOC</th>
<th>Diffusion coefficient in m²/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>$8.10 \times 10^{-13}$</td>
</tr>
<tr>
<td>Toluene</td>
<td>$8.10 \times 10^{-13}$</td>
</tr>
<tr>
<td>Ethylbenzene</td>
<td>$5.79 \times 10^{-13}$</td>
</tr>
<tr>
<td>p-Xylene</td>
<td>$8.10 \times 10^{-13}$</td>
</tr>
<tr>
<td>Chloroform</td>
<td>$5.79 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

### Phase 3 – Extraction of air

Concentrations in the source and receptor cells were measured and plotted (initial concentration over measured concentration) against time for the source and receptor volumes of each cell. The results showed that the VOC concentrations in the source gradually decreased while concentrations in the receptor gradually increased over the 48-day testing period. When looking at the graphs of concentrations (initial concentration over measured concentration) against time per individual VOC in the various cells, another trend becomes clear. Figures 14 and 15 show this trend for benzene, but the trend was also the same for the other VOCs (not shown due to article space constraints).

These figures show that concentrations of the VOCs increase more significantly in the receptor volume of Cell A where no air extraction took place, compared to the other cells’ charts that represent various air extraction rates. The concentration versus time output graphs that POLLUTE® produced were combined with the actual laboratory test results and are shown in Figures 16 to 19 for Phase 3 testing, again using only benzene as an example.

For Phase 3 testing the diffusion coefficients obtained through POLLUTE® are given in Table 4 on p 44.

### DISCUSSION

Phase 1 results show that over the 22-day diffusion test period, the VOC concentrations in the source decreased and the VOC concentrations in the receptor increased. The detection of VOC concentrations in the
receptor started on day 8 and increased to between 10% and 30% of the original source concentration at day 22. The VOC concentrations in the source immediately decreased as the VOC sorbed onto the HDPE GM, and gradually decreased over the 22-day testing period to between 5% and 30% of the original source concentration. Measured losses can be attributable to sorption of the VOCs onto items such as the stainless steel cell, the septa, the gaskets or the screw in the filling port, but since great care was taken to limit losses due to sorption to these areas, the most plausible reason for the losses would be due to the sampling process. Phase 1 testing met its objective of proving that the VOCs in question diffuse from the source, through the 2 mm HDPE GM, into the receptor that represents the groundwater, and the diffusion coefficients obtained compare well with those from literature.

Phase 2 results show that over the 86-day diffusion test period, the VOC concentrations in the source decreased and the VOC concentrations in the receptor increased. The detection of VOC concentrations in the receptor were evident from the samples taken on day 5 already, and increased to between 15% and 32% of the original source concentration at day 86. The trend in the data shows an increase in the receptor and a decrease in the source concentrations over time, indicating that diffusion took place across the divide between the source and receptor. The VOC concentrations in the source immediately decreased as the VOC sorbed onto the HDPE GM, and continued to decrease gradually over the 86-day testing period to between 80% and 20% of the original source concentration. It is also evident that the concentrations of chloroform in the source reduced at a slower rate than the other VOCs, indicating that it would take longer for the chloroform in the system to reach equilibrium. It took longer to reach equilibrium in the system than for Phase 1 testing, since the sorption and diffusion process had to take place over two HDPE GMs and the 8 mm air-filled pervious zone. For diffusion to occur through HDPE GM separating the receptor from the pervious zone, the concentration of the VOCs in the pervious zone had to be higher than in the receptor to drive the diffusive process. Phase 2 testing proved that the diffusion of BTEX and chloroform takes place from source to receptor across a divide consisting of two 1 mm HDPE GMs separated by an air-filled pervious zone.

Phase 3 results show that concentrations in the source volumes decreased over the testing time to about 20% of the original source concentration. Chloroform is the
exception and its concentration reduced to about 50% of the initial concentration. This is very similar to the data shown on the source graph of Cell A (no air flow), indicating that the reduction in source concentrations are comparable, regardless of air flow through the pervious zone, and that the assumption to use the same sorption coefficient in the modelling of Phase 2 and Phase 3 work was sound. Results also show that VOC concentrations in the receptor volumes of Cells B, C and D increased over the testing period, indicating that, even with airflow through the system, concentrations of BTEX and chloroform were observed in the receptor. However, the concentrations of the VOCs in the receptor volumes were at most 0.9% (average 0.4%) of the original source concentrations compared to 20% found during Phase 2 tests. This indicates that airflow resulted in diffusion taking place significantly slower. Due to the very low VOC concentrations measured in the receptor volumes of Cells B, C and D, the graphs look slightly distorted and trend identification is difficult. The receptor graphs showing the concentration profile per cell for each individual VOC against time indicate that the concentrations measured in the receptor volumes of Cell A, where airflow was not introduced, is much higher than the concentrations measured in the receptor volumes of Cells B, C and D, again indicating that diffusion took place significantly slower in the cells where airflow was introduced. The aim of Phase 3 was to prove that, by introducing airflow into the pervious zone between the two 1 mm HDPE GMs, the concentration of VOCs in the receptor volume (due to diffusion through the HDPE GM) could be reduced significantly, and the results indicate that this aim was comfortably achieved.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions
Various studies have shown that VOCs can penetrate even the most well designed liners of waste containment facilities to pollute the groundwater. The method of penetration is advection and/or diffusion, with the main contributor to pollution of groundwater beneath landfills being diffusion.

Phase 1 of the tests undertaken for this study showed that BTEX diffuses through a 2 mm HDPE GM over time, with significant concentrations found in the receptor volumes of diffusion test cells specially made for this project. This confirmed studies undertaken by many researchers in the past. Phase 2 testing proved that diffusion of BTEX and
chloroform takes place from a source to a receptor reservoir separated by two 1 mm GMs with an air-filled pervious zone between them. Phase 3 proved that by extracting air through a pervious zone beneath the GM component of a landfill liner, the concentration of VOCs present in the underlying groundwater can be reduced, since the air removed from the system also removes the majority of VOCs. This phase of testing also confirmed that more frequent removal of air further reduces the VOC concentrations in the receptor, thus implying that a constant airflow through a pervious zone in a landfill liner can significantly reduce concentrations of VOCs in the groundwater beneath landfills and waste containment facilities.

**Recommendations**

To add to the work done for this study it is recommended that extraction fluids other than air be used at more frequent rates. This could include the use of a GCL in testing to understand whether continuous hydration of the bentonite in the GCL will benefit the reduction in contaminant transport. Also, the VOCs that were extracted from the system in the tests undertaken for this study were not captured or measured. If the VOCs are removed from beneath the liner, they need to be routed somewhere (releasing them into the atmosphere does not protect the environment). It needs to be further investigated how to trap the VOCs and treat them after removal.

**REFERENCES**


Updated provisions of SANS 10160-4 for steel structures

C.P. Roth, A. Gebremeskel

A forthcoming revision of SANS 10160-4 (Seismic actions and general requirements for buildings) addresses the omission of structural steel design provisions from the standard, as well as contradictions between SANS 10160 and SANS 10162, the steel design standard. This note discusses the background to the proposed provisions.

INTRODUCTION

Basic provisions for the design of buildings and other structures to withstand earthquake loads have been available in the SANS codes since 1989. However, compliance with the requirements has not been vigorously enforced by the authorities and owners over the past two decades. This is largely due to lack of awareness by design engineers and academics. In some cases it has also been caused by skepticism about the level of seismic risk that exists in South Africa.

Despite having relatively low seismicity by global standards, South Africa has one of the most up to date codes in Africa when it comes to design for seismic resistance. Whereas the code addresses the design of concrete and masonry structures adequately, a gap exists when it comes to the design of structures that are framed in structural steel. Moreover, the fact that South African codes are derived from different code cultures, namely European and North American, at times causes confusion when attempting to design structures for seismic resistance.

Efforts have been made by the Southern African Institute of Steel Construction (SAISC), in collaboration with the South African Bureau of Standards TC98 to address the gap described above. The proposed provisions for the design of steel structures to resist seismic loading are expected to be published in a forthcoming revision of SANS 10160-4 (Seismic actions and general requirements for buildings). This note discusses the background to these provisions, for the benefit of practising engineers and future code developers.

CURRENT CODE PROVISIONS

Current seismic design practice allows structures to dissipate energy by nonlinear material behaviour during major seismic events. In order to avoid undertaking nonlinear dynamic analysis, the codes allow a designer to do a linear elastic analysis and reduce the resulting forces by a behaviour factor to reflect the energy dissipated by nonlinear behaviour. The value of the behaviour factor depends on the material type and structural system used. Structures with more inherent ductility and energy dissipation capacity have larger behaviour factors, which allow greater reduction in design forces.

The original SANS 10160-4:2010 (SANS 2010) and its first amendment SANS 10160-4:2011 (SANS 2011a) both allow a behaviour factor $q$ of 5.0 for ordinary braced steel frames, and 4.5 for steel frames in a moment-resisting frame system. No specific detailing requirements are specified other than “Detailing rules of SANS 10162-1 and SANS 10162-2 shall apply” and “Also refer to EN 1998-1 for detailing rules of structural steelwork.” This is different to the situation for masonry and reinforced concrete structures, where higher behaviour factors are only allowed for structures complying with certain detailing requirements which are given in Annexes A and B. For example, the behaviour factor for a building frame system with reinforced concrete shear walls increases from 2.0 to 5.0 when specific detailing rules are applied. It is therefore inconsistent for the code to allow relatively high behaviour factors for steel structures without providing specific detailing rules for seismic design in SANS 10160-4.

This inconsistency was recognised when the original SANS 10160-4 was written. Wium (2009) identified quantification and confirmation of the behaviour factor for structural steel as a research need in his discussion of the background to the development of the original SANS 10160-4.

The Eurocode EN 1998-1 (EN 1998) allows behaviour factors in the range of 4.5–5.0 only for steel structures properly analysed and detailed in accordance with the requirements for “Dissipative structural behaviour”. For steel structures where no specific detailing rules are applied, a value of 1.5 is recommended in Section 6 of EN 1998-1.

Keywords: seismic design, code provisions, SANS 10160-4, support structures, steel design
SANS 10160-4 also contains no provisions for seismic steel support frames for heavy rigid objects, such as tanks, bins and similar containers storing liquids, gases and granular materials. These common structures are included in the scope of SANS 10160-4, which covers “industrial structures utilising structural systems similar to those of building structures”. It was felt that provisions for seismic design of such structures would be beneficial.

HARMONY BETWEEN CURRENT CODE PROVISIONS

In addition to the omission of structural steel design provisions in SANS 10160-4 addressed above, there appears to be a contradiction between SANS 10160 and SANS 10162. The SAISC proposed modifications to both SANS 10160-4 and SANS 10162-1 (SANS 2011b) in order to resolve this contradiction while addressing the omission.

As stated previously, no specific detailing requirements are specified in SANS 10160-4 other than references to SANS 10162-1, SANS 10162-2 (SANS 2011c) and EN 1998-1. However, Clause 27 of SANS 10162-1:2011 refers the reader to Canadian standard CSA S16 (CSA S16 2014) for seismic design of steel structures. This contradiction has arisen because SANS 10162-1 is based on Canadian standards, while SANS 10160 is primarily based on European standards.

For steel structures where no specific detailing rules are applied, a behaviour factor of 1.5 is recommended in CSA S16. This value can be raised to 1.95 in some cases. These Canadian codes in turn rely on AISC 360/341/358 (AISC 360/341/358 2010) and the associated American standards ASCE 7 (ASCE 7 2010) (referred to as “ASCE 7” in the remainder of the note) and IBC (IBC 2015) to form the basis for their structural steel seismic design provisions.

ASCE 7 in turn allows the use of a behaviour factor of up to 3 for structural steel systems that are not specifically detailed to achieve high ductility. This level of confidence in the inherent ductility of typical steel structures is borne out by a century of experience in the western parts of the United States.

The dominance of American codes in general when it comes to seismic design can be attributed to the efforts of the Federal Emergency Management Agency in extensively collecting, analysing and standardising seismic response data over the past several decades. As such South Africa, Canada and Europe continue to look to American approaches and standards as a basis for their own standards.

PROPOSED CODE FOR BUILDINGS

The intention of the proposed code is not to provide steel detailing guidelines, but rather to deal more rationally with steel structures not specifically detailed for seismic resistance. A conservative option would be to use a behaviour factor of 1.5 in SANS 10160-4 for such steel structures, as recommended in EN 1998-1 and CSA S16. However, it is clear from the discussion above that such conservatism is not warranted when there is sufficient experience and data to support the more economical behaviour factors that are used in American codes such as ASCE 7.

Table 12.2-1 of ASCE 7 defines Response Modification Coefficients, Overstrength Factors and Deflection Amplification Factors for various types of structural systems. The Response Modification Coefficient is equivalent to the behaviour factor in SANS 10160-4. A value of 3.0 is given for “Steel systems not specifically detailed for seismic resistance, excluding cantilever column systems”, for Seismic Design Categories B and C.

Cantilever column systems resist lateral forces solely by the columns acting as cantilevers supported at the base. Their exclusion in ASCE 7 is not relevant to the proposed code, as SANS 10160-4 Table 4 only includes braced and moment-resisting frames for steel structures.

To relate Seismic Design Categories B or C to SANS 10160-4 terminology, the clauses must be examined in more detail. Table 11.6-1 of ASCE defines the Seismic Design Categories in terms of Risk Categories I–IV and $SDS$, the “design, 5 percent damped, spectral response acceleration parameter at short periods”. In SANS 10160-4, $SDS$ is equivalent to $S_d(T)$, $T_B \leq T < T_C$ with $q = 1$, which is graphically the value of the flat portion of the design response spectrum in Figure 2. A structure in Risk Categories I–III falls under Seismic Design Categories B or C for $SDS < 0.50$, and a structure in Risk Category IV falls under Seismic Design Categories B or C for $SDS < 0.33$. These Risk Categories roughly correspond to the Importance Classes in Table 3 of SANS 10160-4. While there are some differences between classes II and III in the two codes, the distinction between I–III on one hand and IV on the other hand is the same in SANS 10160 and ASCE 7.

In seismic zone 1 as defined in SANS 10160-4, $a_\mu = 0.1$ g and the resulting $S_d(T)$ for $T_B \leq T < T_C$ is shown in Table 1 for the four different ground types. However, a more accurate comparison should include the effect of the redundancy factor $\rho$ of clause 7.4, which can be taken as 1.2. This minimum value of the factor was included to provide an additional factor to increase the seismic forces, as the specified value for $a_\mu$, 0.1 g, is lower than the values on the latest seismic hazard maps (Wium 2010). Values of $\rho S_d$ are also included in Table 1.

<table>
<thead>
<tr>
<th>Ground type</th>
<th>$S_d$</th>
<th>$\rho S_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.36</td>
</tr>
<tr>
<td>3</td>
<td>0.29</td>
<td>0.35</td>
</tr>
<tr>
<td>4</td>
<td>0.34</td>
<td>0.41</td>
</tr>
</tbody>
</table>

It can be seen that if $\rho$ is excluded, the maximum value of $S_d$ is 0.34, which is well under the 0.5 limit for classes I–III to fall in Design Categories B or C, and very close to the 0.33 limit for class IV. It may be argued that essentially all classes fall in Design Categories B or C, so the value of 3.0 suggested by ASCE 7 is applicable.

If $\rho$ is included, then the maximum value of $\rho S_d$ is 0.41, so classes I–III still fall in Design Categories B or C. Class IV does not fall in Design Categories B or C, as 0.41 exceeds 0.33.

As the emphasis in the SANS 10160-4 code is more on instilling correct structural concepts than on extensive calculations (Wium 2010), it was concluded that the vast majority of structures fall in Design Categories B and C, though possibly more consideration should be given to class IV structures that are founded on very loose soil and a lower behaviour factor should be prescribed in such cases. As a behaviour factor of 3.0 is already more conservative and realistic than the existing code values, it was decided to adopt this value until more research on South African conditions becomes available.

With $q = 3.0$, the inelastic displacement $d_e$ according to SANS 10160-4 clause 9.2 would be $0.7q \times d_e = 2.1 \times d_e$. From the Deflection Amplification Factor given in Table 12.2-1, ASCE 7 prescribes $3.0 \times d_e$. For consistency with ASCE 7 it is recommended that $d_e = 3.0 \times d_e$ be specified in D.2.2 of the proposed code. The factor in ASCE 7 can be reduced by the redundancy and importance factors in certain cases, but it was felt that it is conservative and simple to leave it as 3.0.

Clause D.2.3 of the proposed code deals with elements that transfer load from floor or roof diaphragms to the vertical-lateral force resisting system and is based on clause 12.10.2 of ASCE 7, which requires “collector elements and their connections to vertical elements” to be designed for seismic forces including the Overstrength Factor, which is given as 3 in Table 12.2-1. Clause 12.10.2 of ASCE 7 applies...
to structures in Design Category C, and not to Category B, but no distinction is made in the proposed code to keep it simple and conservative.

**SUPPORT STRUCTURES FOR HEAVY RIGID OBJECTS**

Guidelines for steel support structures for heavy rigid objects are included in clause D.3. They are based on section 15.3 of ASCE 7, “Non-building structures supported by other structures” from which the definition of rigid objects as “objects that have a fundamental period of less than 0.06 s and weigh over 25% of the combined weight of the object and supporting structure” is taken. Values of the Response Modification Coefficients, Overstrength Factors and Deflection Amplification Factors used in clauses D.3.2, D.3.3 and D.3.4 are from Table 15.4-2.

According to section 15.3.2, “The supporting structure shall be designed in accordance with the requirements of Chapter 12 or Section 15.5 as appropriate, and the R value of the combined system is permitted to be taken as the R value of the supporting structural system.” The R value obtained from Table 15.4-2 for “Elevated tanks, vessels, bins or hoppers” can cover the vast majority of support structures that are designed in South Africa.

Clause D.3.4 deals with connections between the object and the support structure and is based on section 15.7.3 of ASCE 7, with some modifications to the language to make it easier to follow in the SANS context. In ASCE 7 the factor by which forces are to be multiplied is given as the “overstrength factor”, which is given as 2 in Table 15.4-2.

Clause D.3.5 deals with a heavy rigid object on grillage beams and is based on section 15.5.5 of ASCE 7, also with some modifications to the language to make it easier to follow.

**CONCLUSION**

A forthcoming revision of SANS 10160-4 addresses the omission of structural steel design provisions from the standard, as well as contradictions between SANS 10160 and SANS 10162, the steel design standard. The proposed revision is based on the ASCE 7 standard. Relevant parts of ASCE 7 have been adapted for the new SANS standard.

**REFERENCES**


The Journal of the South African Institution of Civil Engineering is published quarterly in March, June, September and December. Articles submitted for publication are reviewed by a panel of referees under the guidance of the SAICE Journal Editorial Panel. The journal publishes research papers covering all the disciplines of civil engineering (structural, geotechnical, railway, coastal/marine, water, construction, environmental, municipal, transportation) and associated topics that are relevant to the civil engineering profession, and that preferably have particular relevance to civil engineering in southern Africa and the African continent.

When preparing articles for publication, authors should please take note of the following and comply with the guidelines as set out:

CLASSIFICATION OF ARTICLES CONSIDERED FOR PUBLICATION

- Technical papers are well-researched, in-depth, fully referenced technical articles not exceeding 6 000 words in length (excluding tables, illustrations and the list of references). Related papers that deal with ‘softer sciences’ (e.g. education, social upliftment, etc.) are accepted if they are of a technical nature and of particular interest to the civil engineering profession. The latter type of paper will be subject not only to peer-review by civil engineers, but also to review by non-engineering specialists in the field covered by the paper.

- Technical notes are short, fully referenced technical articles that do not exceed 2 000 words. A typical technical note will have limited scope often dealing with a single technical issue of particular importance to civil engineering.

- Review papers are considered for publication as either technical papers or technical notes on condition that they are the original work of the author and will assist the reader with the understanding, interpreting or applying of the subject under review. A review paper must contain criteria by which the work under review was evaluated, and contribute by synthesising the information and drawing new conclusions from the dissemination of the previously published work.

- Discussion on published articles is welcomed up to six months after publication. The length of discussion contributions is limited to 1 500 words. Where appropriate, discussion contributions will be subject to the normal reviewing process and will be forwarded to the authors of the original article for reply.

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- Language: Manuscripts should preferably be presented in English, as the journal is distributed internationally. Articles submitted in any of the other official South African languages should be accompanied by an expanded abstract in English.

- Original work: Papers and technical notes must be original contributions. Authors must affirm that submitted material has not been published previously, is not under consideration for publication elsewhere and will not be submitted elsewhere while under consideration by the SAICE Journal Editorial Panel. It is the responsibility of the authors to ensure that publication of any paper in the journal will not constitute a breach of any agreement or the transgression of any law. The corresponding author should confirm that all co-authors have read and approved the manuscript and accept these conditions. Authors are responsible for obtaining permission to publish experimental data and other information that may be confidential or sensitive. Authors are also responsible for obtaining permission from copyright owners when reproducing material that has been published elsewhere. Proof of such permission must be supplied.

SUBMISSION PROCEDURES AND REQUIRED FORMAT

- Electronic submission: Manuscripts should be e-mailed to the editor at veredene@saice.org.za. File sizes should not exceed 4 MB per e-mail – figures may for example be sent one by one or in groups not larger than 4 MB. Manuscripts should not be sent in PDF format as this precludes reviewing of papers per track changes.

- Format: Manuscripts should be prepared in MS Word and presented in double line spacing, single column layout with 25 mm wide margins. Line numbers must be applied to the whole document. All pages should bear the authors’ names and be numbered at the bottom of the page. With the exception of tables and figures (see below) the document should be typed in Times New Roman 12 pt font. Contributions should be accompanied by an abstract of not more than 200 words.

- First page: The first page of the manuscript should include the title of the paper, the number of words of the main text (i.e. excluding figures, tables and the list of references), the initials and surnames of the authors, professional status (if applicable), SAICE affiliation (Member, Fellow, Visitor, etc), telephone numbers (landline and mobile), and e-mail and postal addresses. The name of the corresponding author should be underlined. Five keywords should be suggested.

- Figures, tables, photos and illustrations: These should preferably be submitted in colour, as the journal is a full-colour publication.

  - Their positions should be clearly marked in the text as follows: [Insert Figure 1]

  - Figures, tables, photos, illustrations and equations should be numbered consecutively and should appear in the text directly after they have been referred to for the first time.

  - Illustrations must be accompanied by appropriate captions. Captions for tables should appear above the table. All other captions should appear below the illustration (figures, graphs, photos).

  - Only those figures and photographs essential to the understanding of the text should be included. All illustrations should be referred to in the text.

  - Figures should be produced using computer graphics. Hand-drafted figures will not be accepted. Lettering on figures should be equivalent to a Times New Roman 9 pt font or slightly larger (up to 12 pt) if desired. Lettering smaller than 9 pt is not acceptable.

  - Tables should be typed in Times New Roman 9 pt font. They should not duplicate information already given in the text, nor contain material that would be better presented graphically. Tabular matter should be as simple as possible, with brief column headings and a minimum number of columns.

- Mathematical expressions and presentation of symbols:

  - Equations should be presented in a clear form which can easily be read by non-mathematicians. Each equation should appear on a separate line and should be numbered consecutively.

  - Symbols should preferably reflect those used in Microsoft Word Equation Editor or Mathtype, or should be typed using the Times New Roman symbol set.

  - Variables in equations (x, y, z, etc) as well as lower case Greek letters should be presented in italics. Numbers (digits), upper case Greek letters, symbols of metric measurement units (m for metres, s for seconds, etc) and mathematical/trigonometrical functions (such as sin, cos and tan) are not written in italics, but in upright type (Roman). Variables and symbols used in the body of the text should match the format used in the equations, i.e. upright or italics, whichever is applicable.

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  - Decimal commas may be used, but decimal points are preferred.

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- Headings: Sections and paragraphs should not be numbered. The following hierarchy of headings should be followed:

  - HEADING OF MAIN SECTION

  - Heading of subsection

  - Heading of sub-subsection

- References: References should follow the Harvard system. The format of text citations should be as follows: “Jones (1999) discovered that...” or “recent results (Brown & Carter 1985; Green et al 1999) indicated that...”

- Footnotes, trade names, acronyms, abbreviations: These should be avoided. If acronyms are used, they should be defined when they first appear in the text. Do not use full stops after abbreviations or acronyms.

- Return of amended papers: Papers requiring amendments will be accepted up to six months after the referee reports have been sent to authors, after which the paper will be withdrawn from the system.

FINAL ARTICLE

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- Photos of authors: The final corrected version of the paper should be accompanied by recent, high-resolution head and shoulders colour photographs and a profile not exceeding 100 words for each of the authors.

- Proofs: First proofs of papers will be sent to authors in PDF format for verification before publication. No major rewrites will be allowed, only essential minor corrections.
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The SAICE Journal Editorial Panel would like to thank the persons listed below, all of whom served as referees during 2016. The quality of our journal is not only a reflection of the level of expertise of participating authors, but certainly also of the high standard set by our referees.

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