The effect of global and partial factors of safety in bearing capacity calculations

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At first sight the change from a global safety factor to partial safety factors seems quite sensible and straightforward. This may well be so where the analyses to be used show a linear relationship between the parameters to be factored and the final result. However, in geotechnical analyses, this is frequently not the case and the application of partial safety factors to \( k \) and \( \phi \) will have further effects on other factors such as earth pressure coefficients and bearing capacity factors. This study has looked at one category of geotechnical analyses, namely bearing capacity analyses and has shown that the overall margin of safety, comparing actions with resistances, when using partial safety factors varies from about 1.5 to about 4. While the use of partial safety factors can be argued to be better than the use of a single global safety factor, this large variation in overall margins of safety must be a cause for concern, given that the 'traditional' use of a global safety factor of about 3 has proved adequate in limiting failures to a frequency acceptable to society.

INTRODUCTION

The limit state method as prescribed by Eurocode 7 (ENV 1997-1: 1992) requires the use of a characteristic value, eg \( \tan \phi' \), for the relevant soil strength property. This value is used in combination with a partial factor to determine the design value for calculations at the limit state. In contrast, the mean value of the soil property, eg \( \tan \phi' \), is commonly used in the Global Factor of Safety (GFS) method. Eurocode 7 (EC7) does not, however, give any definitive means of finding the characteristic value. The selection of this value by statistical means is discussed in section 2.4.3(6) of EC7, as is the use of a 'cautious estimate of the value affecting the occurrence of the limit state' in 2.4.3(5). It has been suggested that judicious application of engineering judgement is required here, and in fact this is one of the few places in the code where there is scope for judgement at all. In giving consideration to changing from the GFS method to the EC7 approach it is important to know whether or not the new approach will lead to designs that are essentially similar to those that we currently produce. If this is not the case, then we need to determine why the difference occurs and whether or not the change is for the better. This paper examines the effect on the foundation size of using the mean value, \( \tan \phi' \), in Global Factor of Safety designs and the mean minus one half standard deviation, \( \tan \phi'_{\text{mean}} - \frac{1}{2} \sigma \text{SD} \), as the characteristic value, \( \phi' \), in Partial Safety Factor (PSF) designs.

Mortensen, quoted in DGI (1993), has investigated variations in foundation area for a column footing for Partial Safety Factor design, charting changes that result from modifications to the partial factors that have been made over the years. In general there has been a decrease in the required size due to reductions in the partial factors, as confidence has increased owing to the absence of reported failures. Over a 20-year period since the codification, in 1965, of Limit State Design and the formalised usage of partial factors there has been a 14% reduction in the required foundation area.

Other investigations of the effects of partial factors on footing size include Bengtsson et al (1993), who compared foundations designed using five Limit State codes, manuals or procedures. They found that Eurocode 7 gave foundation pressures within the range given by the other four. The ratio of highest foundation pressure to the lowest foundation pressure was about 2 for loose sands and about 3 for dense sands.

Hartikainen and Heinonen (1993) compared three partial factor procedures with the total safety factor method for rectangular footings. They found that the total safety factors for the footings required by the partial factor methods were in general above 2, but the range in sizes and total safety factors was very wide.

Orr (1993) considered a pad footing on Dublin black boulder clay and noted differences between the allowable bearing pressure for a GFS of 3 and the design bearing resistance using Eurocode 7. It is not clear, however, how the partial factors for the actions should be, or were, included in this analysis.

No investigations into the effect of the use of partial factors instead of global safety factors appear to have been undertaken for South African applications. This investigation attempts to initiate critical investigation of the likely effects on our foundations, starting with the relatively simple case of spread footings.

SHEAR STRENGTH VALUES

Eurocode 7 proposals

Eurocode 7 defines the characteristic value in section 2.4.3.5 as a cautious estimate of the value affecting the occurrence of the limit state. This is perhaps not the clearest of definitions. Clause 2.4.3.2 of the code states that the selection of the characteristic soil and rock properties shall be based on the
results of laboratory and field tests. The code further states that statistical methods may be used to select the characteristic values for the ground property, such that the calculated probability of a worse value of the mean occurring is not greater than 5 per cent. Ovesen (1995) suggested that to obtain the above value, the characteristic value, \( \tan \phi'_{m} \), should be taken as the mean of the test results less half a standard deviation, \( \pm \frac{1}{2} \text{SD} \).

The traditional approach
In designs using a Global Safety Factor, on the other hand, there is little formal guidance on the selection of the appropriate value of shear strength that should be used. We should also note one of the definitions of a Factor of Safety, namely that a Factor of Safety is a number which, when applied in a design process, leads to a frequency of actual failures which is acceptable to society. The selection of the value of the Factor of Safety that provides this acceptable frequency of failures cannot be separated from the way in which the shear strength has been determined. If, for an extreme example, the design shear strength was taken to be half the mean value, then we would find an apparently low Factor of Safety to be adequate.

There are few cases, such as the ‘\( \alpha \) method’ for pile design (API 1982) where there is specific guidance. In the original \( \alpha \)-method, the undrained shear strength to be used was specified as the lower bound value obtained from the results of triaxial compression tests on 2,5 inch driven tube samples. If, instead, the mean value of undrained shear strength had been used in the derivation of this method, then the \( \alpha \) values would necessarily have been lower.

In general, however, designers seem to use a cautious value of the average. This value is used with the generally accepted value of the Factor of Safety and leads to a satisfactorily low occurrence of failures. Let us call this value the ‘appropriate’ value. Before making a change from this approach, which is effectively calibrated by historical evidence from failures (and the lack of failures) to a new approach, we should be clear about the effect that the change will have on our overall margin of safety.

One question that needs to be asked is what would the effect be of using the same value of shear strength as the characteristic value in PSF design and as the ‘appropriate’ value in GFS design. A second question that needs to be examined in parallel with the first is what would the effect be of using the mean minus half a standard deviation for PSF designs as compared to using the appropriate value in GFS design. To do this and to examine in more depth the effect of the way in which these values are determined, a set of analyses has been performed using ranges of soil properties and standard deviations of these properties.

Investigation process
Calculations were performed to compare results from limit state and factor of safety designs for spread foundations on a purely frictional material, i.e. \( \phi' = 0 \), corresponding to a normally consolidated clay or a sand, as well as for a frictional and cohesive material, i.e. \( \phi' > 0 \), corresponding to an overconsolidated clay or a dense clayey sand. The PSF calculations use a characteristic value which is found by reducing the mean value of the shear strength, which was used for the factor of safety calculations, by half an assumed standard deviation, i.e. \( \tan \phi'_{m} = \tan \phi'_{m} - \frac{1}{2} \text{SD} \). The design value, \( \tan \phi'_{d} \), is then obtained by applying a partial safety factor, \( Y_{m} \), to the characteristic value, i.e. \( \tan \phi'_{d} = \tan \phi'_{m} / Y_{m} \). A range of assumed standard deviations from 0 to 12% has been used. The use of a standard deviation of 0% of the mean value is equivalent to using the same value as the appropriate value in the GFS calculations and as the characteristic value in the PSF calculations, i.e. \( \tan \phi'_{d} = \tan \phi'_{m} \). The larger values of standard deviation correspond to data with a larger scatter, or variation, in the test results. The comparison between the two approaches was done by determining the size of a spread footing obtained from each method for the same assumed shear strength data and loading. For any particular set of assumed conditions the larger foundation size will correspond to a greater overall margin of safety.

Basic Procedures Followed
The analytical method chosen is relatively simple, so that the effects of the different methods of shear strength parameter selection are not masked by the complexity of the analyses. Since the same analytical method has been used for PSF and GFS analyses, the particular analytical method should have no impact on the resulting comparison.

It is usual in GFS design to assume a foundation size and check that the Factor of Safety is adequate. The PSF design approach is more suited to being used to determine the foundation size for which the design actions are balanced by the design resistances. In order to follow the latter approach, the width of a square spread foundation was calculated for a range of assumed loads and assumed soil properties by both the GFS method and the PSF method. Two values of FOS and three values of standard deviation were used.

Scope
The PSF calculations used a characteristic value, which was found by reducing the assumed average coefficient of friction, \( \phi'_{m} \), and the cohesion, \( c' \), by half the assumed standard deviation, e.g. if the assumed value of \( \tan \phi'_{m} = 0.4 \), then the characteristic value, \( \tan \phi'_{d} \), is \( 0.4 - \frac{1}{2} \text{SD} \).

The GFS method used the assumed average value without any modification, e.g. if the assumed value of \( \tan \phi'_{m} = 0.4 \), then this is the value used as the appropriate value in the Factor of Safety method.

The following actions, soil properties and standard deviations were used in the first set of analyses that will be discussed.

- Mean angle of friction, \( \phi'_{m} \), of 20° and 35°
- Cohesion of zero
- A saturated unit weight of 20 kN/m³
- A permanent vertical action of 1 000 kN or 2 000 kN
- A variable horizontal action ranging from 0 to 12% of the permanent vertical action in steps of 4%
- The standard deviation was taken as 0%, 6% or 12% of the assumed average value of the coefficient of friction.

In the set of analyses which will be discussed later, values of cohesion, \( c' \), were taken to be 0, 15, 30, 45 and 60 kPa, together with \( \phi'_{m} \) values of 20, 25, 30 and 35°. The standard deviation for this set of analyses was set at 6% on both \( c' \) and \( \phi' \) throughout.

The permanent action was assumed to act vertically at the centre of the foundation, while the variable action was assumed to act horizontally at a point 3 m above the base of the foundation. The weight of the foundation was taken to be included in the permanent action, and founding depth was assumed to be 1 m, as shown in figure 1.

![Figure 1 Assumed details of foundation](image-url)
METHOD OF ANALYSIS

The foundation width required to resist the applied actions was calculated using Meyerhof's method (Das 1994:485). Meyerhof's equation for bearing capacity is written in slightly different forms for the PSF calculation and the GFS calculation. The bearing capacity factors, $N_c$, $N_q$, and $N_p$, were obtained using the method given by Whitlow (1990:432). For the purpose of this paper, it is not particularly important which set of equations are used to calculate these factors as long as the same equations are used in both the GFS method and the PSF method. The shape, depth and inclination factors were calculated using the equations given by Das (1994:887). The analyses were based on an eccentric reaction representative of a uniform soil stress acting on a reduced area, the effective area, such that the reaction (which must be equal to the vertical load) multiplied by the eccentricity was equal to the moment generated by the horizontal load.

GFS method

For the GFS method, the safe bearing pressure was calculated using the following equation:

$$ q_{safe} = f_{k} \left( c \cdot N_c F_p F_q F_d + 0.5 \cdot \gamma_s N_q F_p F_q F_d + \gamma_s DN_q F_p F_q F_d \cdot \frac{1}{\phi^2} \right) $$

where $c$ = assumed average value of cohesion $N_c$, $N_q$ and $N_p$ are the bearing capacity factors $F_p$ are the appropriate shape, depth and inclination factors and were obtained using $c^*$, the assumed average value of $q_{safe}$ and the un factored values of the loads.

PSF method

For the PSF approach the equation differs slightly and can be written as:

$$ q_{safe} = c \cdot N_c F_p F_q F_d + 0.5 \cdot \gamma_s N_q F_p F_q F_d + \gamma_s DN_q F_p F_q F_d \cdot \frac{1}{\phi^2} $$

where $c$ = design value of cohesion and the factors in the equation were obtained using the design values of $c^*$, $q^*$ and the design values of the actions.

In the PSF method the design values of actions and soil properties were obtained using the appropriate partial factors from table 2.1 of Eurocode 7. These design values were used to determine the relevant factors for use in the bearing capacity equation.

Three cases are specified in table 2.1 of Eurocode 7. Only cases B and C were considered as case A corresponds to buoyancy problems. The relevant values are reproduced in table 1. For case B the vertical action can be considered to be favourable or unfavourable. These sub-cases are labelled Bf and Bu in this paper.

Sub-cases may sound unreasonable or unnecessary but, for large eccentricities, an increase in the vertical action can reduce the eccentricity sufficiently to reduce the required foundation width. Increasing the characteristic vertical action by a factor of 1.3 for the unfavourable case could lead to a smaller foundation width than the use of a factor of 1 for the favourable case. (This can be seen in table 2b for the 120 and 240 kN horizontal load cases, and extreme care is advised.

<table>
<thead>
<tr>
<th>Loading</th>
<th>GFS method, FOS =</th>
<th>Required foundation widths in metres</th>
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<tr>
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<td>Horizontal</td>
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<tr>
<td>(kN)</td>
<td>(kN)</td>
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<tr>
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<td>0</td>
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<td>4.30</td>
</tr>
<tr>
<td>2 000</td>
<td>240</td>
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Table 2b Foundations widths required by GFS and PSF methods for $\phi = 35^\circ$
In choosing favourable and unfavourable cases, even though Simpson (1984) suggests that engineers will soon realise which case will govern the design under particular circumstances.

**Calculations**

In order to find the required foundation size, a width was assumed and the shape, depth and inclination factors were calculated. The bearing capacity (either the safe bearing capacity for the GFS calculation or the ultimate bearing capacity for the PSF calculation) for this foundation width was calculated, and compared with the actual bearing stress or the design bearing stress exerted by the foundation on the ground. The foundation width was then adjusted accordingly. This process was repeated until the bearing capacity was equal to the uniform bearing stress acting on the effective foundation area. For the GFS method this gives the foundation width that will give the chosen Factor of Safety while for the PSF method this gives the foundation width corresponding to the specified partial factors.

Clause 6.5.4 (2) in Eurocode 7 requires that where the eccentricity of the load is greater than one third of the width of a rectangular footing, certain precautions should be taken. The clause also suggests that unless special care is taken during the works, differences in the dimensions of the footing of up to 0.1 m should be considered. The code is unclear about what is meant by this clause, but it appears from the context that it refers to the location of the foundation edge and that 0.1 m should be added to the calculated width. There is no suggestion that this additional width should be related to the actual foundation width. Therefore, where the eccentricity of the loads in the calculations following was greater than one third of the width of the footing, 0.1 m was added to the size of the base in each direction.

**RESULTS**

The results from the first set of calculations are presented in Table 2. Table 2a shows the foundation widths calculated for an assumed average value of $\psi_m^c$ of 20°. The values given in the body of the table are the foundation widths, in metres, that have been calculated for the appropriate loading and method. For example, shown in italics, a vertical action of 1 000 kN with no horizontal action required a foundation width of 2,71 m for a Factor of Safety of 2,0 or a foundation width of 2,54 m for case C with a standard deviation of 0°.

The smaller width required by the PSF method for this set of conditions indicates that the overall margin of safety provided by the PSF method is less than that provided by the GFS method with a Factor of Safety of 2. This corresponds to the case where the same value of $\psi_m$ is used as the characteristic value in theFoundation widths determined by the PSF approach correspond to Factors of Safety ranging from about 2,2 for a Standard Deviation of 0% to about 2,5 for a Standard Deviation of 12%, where they were below 2 for no horizontal action. We can thus see that the margin of safety has increased as the live load has increased. The equivalent factors of safety for the $\psi_m = 35°$ case are higher and lie well above 3 for all values of the assumed standard deviation. These observations warrant further investigation but do not form part of the subject of this paper.

The results of the second set of calculations are presented in Table 3. The data shows that the PSF method can be either more or less conservative than the GFS method. For low friction angles and low cohesion, the PSF method produces foundations with horizontal action the equivalent factor of safety, for the three assumed values of standard deviation, ranges from just below 2,5 to very nearly 3. The implications of this are that for low friction angles, corresponding to normally consolidated clays, the PSF design will be less conservative than a typical GFS design with a Factor of Safety of 2 or 2,5, but for high friction angles, corresponding to sands, the PSF design will be more conservative than a typical GFS design with a Factor of Safety of 2 or 2,5. This apparent lack of consistency is a possible cause for concern.

Returning to Table 2a and 2b a similar set of observations can be made for the vertical action of 2 000 kN and no horizontal action. This is not surprising since there is no major difference between the 1 000 to 2 000 kN vertical action cases.

<table>
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<th>$\psi_m$</th>
<th>Loading</th>
<th>Required foundation widths in metres</th>
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<td></td>
<td>$V_1$ (kN)</td>
<td>$H_1$ (kN)</td>
</tr>
<tr>
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</tr>
<tr>
<td>20°</td>
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<td>1 000</td>
<td>0</td>
</tr>
<tr>
<td>35°</td>
<td>1 000</td>
<td>120</td>
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<th>$\psi_{m}$</th>
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<th>Reduction factor, $N_{N_{GFS}}$ for GFS</th>
<th>$N_{GFS}$</th>
<th>$N_{GFS}$ for PSF</th>
<th>$N_{GFS}$ for GFS</th>
<th>$N_{GFS}$ for PSF</th>
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<th>$N_{GFS}$ for PSF</th>
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<th>$N_{GFS}$ for PSF</th>
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<td>20°</td>
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<td>35°</td>
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<td>0.560</td>
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<td>33.3</td>
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<td>16.1</td>
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### DISCUSSION

This variation in the overall margin of safety does not seem to be a logical consequence of the change of method, unless the method which has been applied in the past – ie the GFS method – is not reliable. However, since we have depended on it for many years with sufficiently few failures, this consequence should be examined in more detail, particularly for the lower friction angles where the PSF appears to be less conservative.

Giving a little more depth to the study leads to the observation that this apparently anomalous situation results from the different point of application of the factors combined with the strongly non-linear relationship between $\psi$ and the bearing capacity factors. Table 4 shows the reduction in the $N_{GFS}$ and $N_{PSF}$ bearing capacity factors for $\psi_{m}$ values in 20° and 35°.

In both PSF cases $\tan\psi_{m}$ has been reduced by the partial factor of 1.25. For the 20° case the corresponding reductions in the bearing capacity factors are by factors of about 1.4 and 2, respectively, while for 35° the reductions are by factors of 2 and 2.5, respectively. If we are satisfied that the uncertainty that we are trying to cater for is in the soil property, ie $\psi_{m}$, then this is indeed the correct way to reduce the calculated bearing capacity. By contrast, the GFS approach, where the factor is applied after the determination of the bearing capacity factors, provides a constant reduction in the bearing capacity equivalent to the value of the PSF. In other words, the GFS method makes less allowance for uncertainty in $\tan\psi_{m}$ for high values of $\psi$ than it does for low values of $\psi$. This appears less justifiable than the PSF approach where the allowance for uncertainty in $\tan\psi_{m}$ remains constant over the range of $\psi$.

The effect of an increase in the cohesion which gives a larger increment in the foundation width is perhaps surprising since the partial factor of safety applied to $c$ is 1.6 and there are no non-linear relationships between $c$ and the bearing capacity. The effect of $c$ should therefore be uniform across the range of $\psi$. It would be expected that the inclusion of $c$ should lead to smaller foundations than the GFS method, since the maximum overall factor applied through $\psi$, including the partial factor for the actions, is 1.6 x 1.5 = 2.4 for the variable actions.

However, when determining $N_{GFS}$ the effect of $\psi$ cannot be ignored. For the GFS case with $\psi = 35°$, the value of $N_{GFS}$ is 46.1, while for the PSF case with $\psi = \arctan(\tan 35\times(1/6/9/6/1/2.5)) = 27.7°$ the value of $N_{GFS}$ is 25.2, which gives an additional reduction factor of 1.8. Combining this with the factor of 2.4 above gives an overall factor of 4.3, which is a larger reduction than we have customarily used in the GFS approach. Finally, the different values of $\psi$ and $\psi$ will affect the values of the shape, depth and inclination factors, giving a different apparent overall margin of safety for each case.

It is of interest to note that case C governs the foundation width throughout this study but this should not be assumed to be universally true.

### SUMMARY

This rather brief investigation shows that there is a fundamental difference between the traditional GFS approach and the proposed PSF approach in Eurocode 7. Three important aspects need to be given consideration by engineers who may choose to accept Eurocode 7 for use in South Africa.

First, it has been shown that the use of Eurocode 7 with the partial factors that are given in the code will lead to spread foundations that are smaller or larger than those that would be obtained from traditional GFS designs with Factors of Safety in the range 2 to 3. The foundation widths obtained from the PSF approach correspond to Factors of Safety between 1.5 and 3 for the GFS method. Alternatively, we may consider the results to show foundation widths for the PSF method ranging from 10% smaller to 40% larger than those required by the traditional GFS approach. It may be argued that spread foundations are generally governed not by strength considerations but by settlement. This tends to be true for normally consolidated clays and loose sands but is not generally true for overconsolidated clays, dense sands or bonded residual soils. The question that must be answered by the profession is: Are these smaller foundations still safe enough to avoid an excessive frequency of failures under South African conditions? The traditional approach has served adequately and could thus be considered to have been informally "calibrated" against the observed rate of failures. However, it is also possible that the traditional method is in fact over-conservative in some conditions and under-conservative in others. Thorough evaluation of this will require considerable more analysis and probably some model or prototype testing.

Second, it has been shown that the origin of the margin of safety is not the same for the two approaches. We have traditionally applied the margin of safety between the bearing capacity and the applied loads. Eurocode 7 appifies a constant factor to the shear strength parameters and this leads to a variation in the margin of safety between the bearing capacity and the applied actions as the coefficient of friction varies. Here again, our traditional method may be considered to have a 'calibrated' status and there has been no reported inconsistency between performance of foundations on soils with high coefficients of friction as compared to those on soils with low coefficients of friction. However, a question that must be asked is: Is the uncertainty really in the coefficient of friction? Do we implicitly rely on an excess of strength to incorporate uncertainties in many other aspects, as described by Beal (1979), for a relatively well-controlled engineering field such as Structural Design?

Third, it has been shown that the inclusion of cohesion in the determination of foundation sizes by the PSF method will in many cases lead to larger foundations than we would have required using the GFS method, owing to the effect of $\psi$ on the $N_{GFS}$ value.

### References


DGI 1993. Principles used in Denmark for the assessment of partial factors of safety in Geotechnical Engineering. DGI Report 1,1600816D to Danish Standard.


