Generic formulation of member strength as a step towards a unified structural code

N W Dekker, W M G Burdzik and V Marshall

The structural design codes particular to the various structural materials appear to have developed in isolation. Structural mechanics, methods of analysis and limiting-stress-based calculations are common to all materials. In this paper the authors emphasise commonalities in the formulation of member resistance for various structural materials and suggest the possibility of a unified structural design code. However, the authors do not attempt to propose a complete revised, unified code. Serviceability issues in particular are not dealt with, although the generic formulation may be extended to all code requirements. By considering the most common structural materials, some remarkable similarities between structural members of different materials are demonstrated. Load-bearing masonry is not considered, as it falls outside the scope of expertise of the authors.

CURRENT STATUS AND RECENT HISTORY

Design codes essentially consist of a loading code and a number of material specific structural design codes. Considering that the loading code, SANS 10160, 1989 is currently under review, it may well be an opportune moment to consider some aspects of the material specific structural codes.

South African structural codes were ostensibly British based. The main departure from this occurred when the structural steel code, SANS 10162, 1992 (Limit-states), was based upon the Canadian Code, CAN/CSA-S16.1-M89, in order to directly incorporate member resistance equations particular to grade 300 W steel. Prior to this, the lamentable situation existed that engineers were using ULS principles in reinforced concrete, and permissible stress design in structural steel and timber. Clearly, this situation was never conducive to consistent design standards.

Differentiation between the behaviour of ductile and brittle materials, members and structural fasteners is commonly integrated as partial material factors, although it is recommended that current values of partial material factors be retained unchanged.

Ductile and brittle failures

D Differentiation between the behaviour of ductile and brittle materials, members and structural fasteners is commonly integrated as partial material factors, which are the inverse of partial material factors, of 0,9 for ductile failures.
ures and 0.67 for brittle failures are consistent with current design codes in use in South Africa, for example SANS 10162 and SANS 10100.

Laboratory tests show that ductile materials such as steel invariably exhibit a smaller variance in terms of strength than brittle materials such as timber and masonry. This aspect is reflected in the calibration of design codes in terms of the relative values of partial resistance factors. In a generic approach to member and connection resistance it is not necessary to further distinguish between brittle and ductile materials.

**Fundamentals of resistance formulation of structural members**

It will be noted that the proposed generic formulation of member and connector resistance will yield exactly the same answers as the current formulation contained in the various material-specific codes. The generic re-formulation essentially sets out to achieve consistency in notation and structural parameters.

Prior to the introduction of a generic formulation of member resistance the following principles should be considered:

- The mode of failure should be clearly reflected in the formulation.
- Resistance formulation should clearly reflect the differences in mechanical properties of the various structural materials.

Generally speaking, structural members and connectors fail by a combination of individual load effects consisting of axial force, bending moment and shear force.

The design of structural connections is governed by basic principles of static analysis and calibrated resistance values of the components in the connection.

Factors influencing the resistance of members to primary load effects may be summarised as shown in Table 1.

As will be shown subsequently, all the parameters referred to in Table 1 are at present contained in the formulation of member and connection resistances in existing design codes.

The proposed generic formulation is not intended to replace empirical design rules. It is believed that such rules can be re-formulated, however.

**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ac</td>
<td>effective compression area</td>
</tr>
<tr>
<td>At</td>
<td>gross area</td>
</tr>
<tr>
<td>Ak</td>
<td>net area</td>
</tr>
<tr>
<td>Ai</td>
<td>area of reinforcement</td>
</tr>
<tr>
<td>Ao</td>
<td>effective tensile area</td>
</tr>
<tr>
<td>Ap</td>
<td>applied axial force</td>
</tr>
<tr>
<td>Ar</td>
<td>factored resistance of a compression member</td>
</tr>
<tr>
<td>D</td>
<td>dead load effect</td>
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<tr>
<td>g</td>
<td>shear modulus</td>
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<tr>
<td>I</td>
<td>St Venant torsional constant</td>
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<tr>
<td>L</td>
<td>load effect</td>
</tr>
<tr>
<td>Mr</td>
<td>elastic buckling moment</td>
</tr>
<tr>
<td>Mm</td>
<td>applied moment about the x-axis</td>
</tr>
<tr>
<td>My</td>
<td>applied moment about the y-axis</td>
</tr>
<tr>
<td>Mf</td>
<td>factored resistance of a flexural member</td>
</tr>
<tr>
<td>R</td>
<td>yield moment in a steel beam</td>
</tr>
<tr>
<td>S</td>
<td>member resistance</td>
</tr>
<tr>
<td>Sx</td>
<td>strength</td>
</tr>
<tr>
<td>Ty</td>
<td>geometric section property</td>
</tr>
<tr>
<td>U1,2</td>
<td>moment amplification terms about x-axis and y-axis</td>
</tr>
<tr>
<td>V</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>Vc</td>
<td>factored member shear resistance</td>
</tr>
<tr>
<td>Zo</td>
<td>section modulus in bending</td>
</tr>
<tr>
<td>f</td>
<td>width of member</td>
</tr>
<tr>
<td>fb</td>
<td>limiting material stress</td>
</tr>
<tr>
<td>fL</td>
<td>limiting bending stress</td>
</tr>
<tr>
<td>fE</td>
<td>effective compression strength</td>
</tr>
<tr>
<td>fT</td>
<td>effective tensile strength</td>
</tr>
<tr>
<td>fu</td>
<td>ultimate stress</td>
</tr>
<tr>
<td>fy</td>
<td>yield stress</td>
</tr>
</tbody>
</table>

| h      | height of member |
| n      | exponent of inelastic buckling curve |
| v      | strength of concrete |
| a1,2,3 | exponents of interaction equation |
| β      | safety index |
| h0     | buckling factor |
| b0     | shear buckling factor |
| b1     | confinement ratio |
| φc     | partial material factor |
| γ       | load factors |
| λ      | slenderness ratio |
| α2     | moment gradient correction factor |

**Partial load and resistance factors**

The safety index, β, for two lognormal variates can be described in terms of the mean resistance, $R$, the mean strength, $S$, and the coefficients of variation $V_R$ and $V_S$ respectively and is given by (Leicester 1983):

$$\beta = \frac{\log_{10}(R/S)}{\sqrt{V_R^2 + V_S^2}}$$

The first order, second moment, FOSM, analyses refer to methods in which estimates of the safety index are based solely on mean values and coefficients of variation. (Leicester et al. 1982). For the two-variate problem above the equation may be written as follows:

$$S^{-1}R^{-1} = e^{\alpha + \beta (V_R^2 + V_S^2)}$$

If one now introduces the approximation

$$\sqrt{V_R^2 + V_S^2} = 0.75(V_p + V_S)$$

this leads to: $R e^{-0.75 \beta V_p} = S e^{0.75 \beta V_S}$ which may in turn be written in the form with which most engineers are familiar, namely $\phi R_k = \gamma_k S_k$, where the $k$ refers to the charac-
teristic value, and the material factor, $\phi$ and load factor $\gamma$, by: $\phi = k(\bar{R}/R) e^{-0.75,5 V_k}$ and $\gamma = (S/S_k) e^{-0.75,5 V_k}$ respectively. To this one may add a 'committee factor' so that they take the form of: $\phi = k(\bar{R}/R) e^{-0.75,5 V_k}$ and $\gamma = k(\bar{S}/S_k) e^{-0.75,5 V_k}$.

For more than two variates, for example in the case of a dead load, $D$, and a live load, $L$, the equation of resistance and load can be written as follows:

$$P_F = P_l / (R < D + L)$$

The probability of failure ($P_F$) = probability of resistance, $R$, being less than the dead load effect, $D$, plus the live load effect, $L$. A check with some exact solutions shows that this may be approximated by:

$$\phi R_k = \gamma_D D_k + \gamma_L L_k$$

Where

$$\phi = \frac{R}{R_k} e^{-0.75,5 V_k}$$

$\gamma_D = (D/D_k) e^{-0.75,5 V_k}$

$\gamma_L = (L/L_k) e^{-0.75,5 V_k}$

As before, a committee factor, $k$, may be

### Table 2 Safty index defined by $\phi \beta = P_F$ for unit normal variate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steel</th>
<th>Reinforced concrete</th>
<th>Timber</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_k$</td>
<td>$0.9 f_y / (0.85 f_y)$</td>
<td>$0.87 f_y$</td>
<td>$0.37 \cdot f_y$</td>
</tr>
</tbody>
</table>

### Table 3 Tensile strength parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steel</th>
<th>Reinforced concrete</th>
<th>Timber</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_y$</td>
<td>0.9$f_y$</td>
<td>$0.87 f_y$</td>
<td>$0.37 \cdot f_y$</td>
</tr>
</tbody>
</table>

### Table 4 Parameters for compressive strength of common structural materials

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steel</th>
<th>Reinforced concrete</th>
<th>Timber</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>0.67</td>
<td>1.0/0.91</td>
<td>1.0/0.91</td>
</tr>
</tbody>
</table>

Included in the equations. The safety index for a given probability of failure is given in table 2 for a unit normal variate. This will be used as an illustration to determine the partial material factor for timber-bending elements with a lognormal distribution. Assuming a coefficient of variation of 25% with a mean of 1 and a fifth percentile value of 0.6471. A probability of failure of $10^{-3}$ is required.

$$\beta = 4.75$$

$$\phi = (1/(0.6471)) e^{-0.75.4.75.0.25} = 0.634$$, which is in keeping with the suggested $0.67$ or the $0.68$ given in SANS 10163, 2001.

Calibrated load and resistance factors are included in all design codes. In the South African context, resistance factors have been calibrated to the load factors contained in the loading code, SANS 10160. Resistance factors may, for the purpose of formulating a generic resistance, be directly obtained from the relevant material specific codes.

### GENERIC FORMULATION OF MEMBER STRENGTH

In this paper, the concept of a generic formulation of member resistance will be demonstrated by considering the principal structural members, in order of increasing complexity of the mode of failure. This form of formulation of member strength is independent of the type of material used and the design code used to quantify the limiting strength. The general form of generic formulation can simply be expressed as follows:

$$\text{Member resistance} = (\text{partial material factor} \times \text{limiting material stress}) \times \text{geometric section property}$$

$R = \phi_f S_p$

The limiting stress multiplied by the partial material factor may also be referred to as an effective stress.

### Tension elements – behavioural considerations and generic strength formulation

The following behavioural considerations apply:

- Overall member strength not influenced by instability.
- Geometric section parameter is related to portion of section capable of resisting tensile forces.

Table 3 is a summary of the parameters that determine the tensile strength of three materials under consideration.

### Steel tension elements

The effective tensile area is generally taken as the gross area using the yield strength and the net area using the ultimate strength of the material. The effective tensile strength is taken as the yield strength multiplied by the partial material factor.

### Reinforced concrete tension elements

The full tensile force is assumed to be resisted by the reinforcement in the section, and therefore the effective area equals the area of reinforcement and the effective tensile strength is given by the yield strength multiplied by a partial resistance factor of 0.87.

### Timber tension elements

The effective tensile area is taken as the net area except when the connector has a diameter of less than 8 mm, the gross area may be used. Consistent with current practice, a partial material factor of 0.67 is used. Additional to this, a factor of 0.55 must be applied to convert the compressive yield stress to a tensile stress.

### Compression elements – behavioural considerations and generic strength formulation

The following behavioural considerations apply:

- Member resistance influenced by local and global instability.
- Stable or stocky member strength used as a basis.
- Material strength dependent on degree of confinement.
- The limiting compressive strength of the stable member may be adjusted to
compensate for member instability and the effective cross sectional area may be adjusted to compensate for local instability.

\[ C_s = \beta_s \beta_c A_f \]

The stable or ‘stocky member’ strength is used as a basis for predicting the actual strength and is adjusted by the buckling factor, \( \beta_b \), to compensate for the influence of global instability.

The confinement ratio factor, \( \beta_c \), is dependent on the tensile strength of the material. In the case of materials with high values of tensile strength, the value of \( \beta_b \) may be taken as 1, as the limiting compressive stress would be consistent with a confined compressive strength. In the case of materials possessing low tensile strength, the value of \( \beta_b \) depends on the state of stress to which the particular member is subjected.

In the case of reinforced concrete columns and beams, this factor is commonly taken to equal 0.67, while in the case of concrete hinges this factor can approach a value of unity. The confinement factor may also be used to reflect the influence of local instability in the case of members consisting of ductile materials such as structural steel. The buckling factor, \( \beta_b \), reflects the reduction in the base strength of a stable, or stocky member, due to overall instability of the member, and is obviously a function of the slenderness of the member. For materials having a low ratio of strength to mass, it will generally be found that the value of \( \beta_b \) can be taken as unity.

The reduction factor for slender members commonly applied to reinforced concrete members (clause 4.7.3 of SANS 10100) may, for this purpose, also be classified as a buckling factor.

The effective compression area, \( A_c \), will commonly equal the gross area of the section in the case of members consisting of isotropic materials. In the case of multi-material members such as reinforced concrete members, the effective area in compression is determined by converting the strength contribution of the second material to an equivalent area of the base material.

The effective compression strength, \( f_c \), is obtained by multiplying the nominal or characteristic strength by the appropriate partial material factor. Any particular design code may be used. Table 4 is a summary of the parameters that determine the compressive strength of the three materials under consideration.

**Structural steel compression elements**

A value of 1 is assigned to \( \beta_s \), consistent with class 1 and 2 sections where local buckling does not influence the capacity of the member. In the case of class 3 and 4 sections the value of \( \beta_s \) should be taken as the ratio between the net and gross areas where the net area is calculated using the effective width concept.

In calculating the value of \( \beta_s \), values of \( n = 1.34 \) for non-stress relieved sections and 3.34 for stress-relieved sections may be used, in accordance with current practice.

**Reinforced concrete elements**

In the case of materials possessing low tensile strength, the value of \( \beta_s \) depends on the state of stress to which the particular member is subjected. In the case of reinforced concrete columns and beams, this factor is commonly taken to equal 0.67, while in the case of concrete hinges this factor can approach a value of unity.

The effective area of a reinforced concrete compression member may be expressed in terms of the gross area adjusted for the amount of compression reinforcing in the section.

**Timber compression elements**

A value of 1 is assigned to \( \beta_s \) as the yield stress is based on the local buckling of the fibres. The buckling factor, \( \beta_s \), is dependent on the slenderness ratio. In calculating the value of \( \beta_s \), values of \( n = 1.8 \) (Burdzik 2002) may be used.

**Flexural members - behavioural considerations and generic strength formulation**

The following behavioural considerations apply:

- Resistance of beams may be considered in terms of a compression field and a tension field.
- Behavioural considerations of simple tension and compression members apply equally to beams.
- Flexural resistance may be quantified in terms of an effective section modulus and an effective flexural strength which is lower-bound by the resistance of either the compression or the tension field.

\[ M_t = \beta_p \beta_c Z_p f_c \]

Where \( F \) = shape factor

The approach is similar to that followed in the treatment of compression members. The limiting strength is based upon that of the stocky or stable member, which is then adjusted for instability.

The confinement factor, \( \beta_c \), is also used to reflect the influence of local instability. This application of the factor allows for a uniform expression of the section modulus in bending. It is proposed that \( Z_c \) always be expressed in terms of the fully plastic modulus in bending. Table 5 is a summary of the parameters that determine the bending strength of the three materials under consideration.

**Steel flexural elements**

The confinement factor, \( \beta_c \), may be used to distinguish between class 1 and 2 and higher classification elements. For class 3 members the value of \( \beta_b \) is equal to the inverse of the shape factor and is consistent with an upper-bound resistance value equal to the yield moment, \( M_y \), while for class 4 members the value of \( \beta_b \) should be further adjusted by the ratio of the net section modulus to the gross section modulus using the effective width concept.

The buckling factor, \( \beta_b \), is obtained using the familiar expression but again the second term in brackets is adjusted for class 3 and four members by the inverse of the shape factor.

**Reinforced concrete flexural elements**

The confinement ratio, \( \beta_c \), is set at 0.67 and the buckling factor, \( \beta_b \), at 1. The expression for the section modulus in bending is derived from rectangular (plastic) stress blocks and is formulated in terms of the concrete strength. A partial material factor of 0.67 is used, consistent with common practice. Note that the expression given in table 5 for \( Z_s \) applies to a singly reinforced, under-reinforced section. Similar expressions have been developed for a doubly reinforced section.

**Timber flexural elements**

It is possible to define a plastic section modulus for timber bending elements. However,
one must then be aware that the confinement factor reflects the inability of the material to go fully plastic and would be equal to the inverse of the shape factor. The buckling factor, $\beta_b$, is obtained using the familiar expression.

Beam-columns – behavioural considerations and generic strength formulation

The following behavioural considerations apply:

- The sum of capacity ratios is used as a basis.
- P-delta effects may be determined on the basis of a second-order analysis or by moment amplification.
- The use of the factors $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ allows for a non-linear interaction between the capacity ratios, as is common in the case of reinforced concrete. In the case of structural steel and timber, these values are normally taken as 1.0.

$$\left( \frac{C_\text{f}}{C_\text{e}} \right)^{n_1} + \left( \frac{U_{1x} \cdot M_{x}}{M_{x}} \right)^{n_2} + \left( \frac{U_{1y} \cdot M_{y}}{M_{y}} \right)^{n_3} < 1$$

Alternative:

$$\left( \frac{R_{\text{f}}}{R_{\text{e}}} \right)^{n_1} + \left( \frac{R_{\text{sb}}}{R_{\text{e}}} \right)^{n_2} + \left( \frac{R_{\text{ty}}}{R_{\text{e}}} \right)^{n_3} < 1$$

Table 6 is a summary of the parameters to be used in the interaction equations governing the strength of beam columns.

Reinforced concrete beams

The shear strength of reinforced concrete beams is provided by the combined strength of the concrete and the shear rein-
forcement. Hence, in the case where shear reinforcement is provided in the form of vertical links, the shear strength, \( f_s \), is given by

\[
f_s = V_c + \frac{0.87 f_v}{b_v} \left( \frac{A_{rel}}{x_c} \right)
\]

The various limits placed on the use of the above expressions by clauses 4.3.4.1.2 and 4.3.4.1.2 of SANS 10100 apply equally here.

Since buckling in shear does not occur in normal reinforced concrete beams, \( \beta_{bs} \) is usually set to 1. The effective shear area \( A_v \) is defined as \( b_v d \).

Shear in timber sections

Buckling in shear does not occur in normal rectangular timber beams so that \( \beta_{bs} \) will in most cases be equal to 1. The shear stress is assumed to have a parabolic distribution so the effective shear area is equal to \( 2/3 \) of the nominal cross-sectional area.

\[
V_r = \beta_{bs} A_v f_v
\]

POSSIBLE CODE STRUCTURE USING A GENERIC FORMULATION OF MEMBER STRENGTH

The current code structure is shown diagrammatically in figure 1. The structure is limited to building codes and does not consider specialised structures such as bridges and water-retaining structures. TMH 7, for example, refers the reader to BS 5400 Part 3 for the design of steel bridges. Apart from the common loading code, there is no incentive towards any cross-pollination between the various material-specific structural design codes.

In the case of the possible unified design code, common formulations for the resistance equations are used. The formulations of member resistance are based on the behavioural considerations particular to the mode of failure under consideration. Variations in the resistance equations particular to different structural materials are calibrated using a set of partial material factors reflecting modes of failure such as instability and brittle failure. Figure 2 shows the possible structure for a unified code with material specific provisions.

CONCLUSIONS

Although the discussion in the text has been deliberately limited with regard to the number of structural materials (three) and the number of failure modes (five), it has been demonstrated that a generic formulation of member strength is possible and extremely useful in demonstrating fundamental similarities and differences in the failure modes of structural members. It is of utmost importance that the practising engineer be sensitised to both the similarities and the differences of the structural materials that he/she utilises in the design of structural members. An understanding of the similarities in the failure modes of structural materials can only serve to broaden the scope and understanding of the practising structural engineer, enabling superior cognitive conceptual skills. An understanding of fundamental differences in materials will alert the engineer to the nature of different failure modes.

The generic formulation enables the engineer to seek and identify the principal parameters in ‘foreign’ design codes, leading to an intelligent interpretation and application of design codes.

References


South African National Standards, SANS 10160-1989 (as amended) 1998. The general procedures and loadings to be adopted in the design of buildings.

