The accuracy of two slope stability programs utilising the Bishop and Sarma analysis methods

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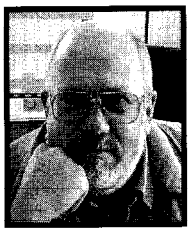

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In 1978 he joined Bruinette Kruger Stoffberg and Hugo as resident engineer for contracts in Caprivi, Pretoria and Owamboland. He returned to the geotechnical department of BKS in Johannesburg and Pretoria before secondment to Offshore Design Services for the foundation design of the Mossgas FA platform.

In 1993 Alan established ARQ Associates with Raimund Miller and Quentin Shaw. At present he heads the geotechnical and project management divisions of ARQ Specialist Engineers.

INTRODUCTION

Slope stability considerations have long captured the attention of geo-practitioners. Fellenius (1936) initially introduced his so-called 'ordinary method of slices' (OMS) for the prediction of earth dam stability. Some 20 years later, Bishop (1955) proposed a method that, despite an assumption of interslice forces, is still used with much success today.[1]

Bishop's formulations, however, suffer from the constraint that internal loads and/or external forces cannot theoretically be included in the computations without altering the assumptions made in the solution for the factor of safety.

This facet imposes a major constraint when external and/or internal loads are present in real-life problems, eg loads at the top of the slope generated by vehicles on road or rails, anchors utilised for stability, or geosynthetics for reinforcement.

This paper presents results of analyses conducted whereby Sarma's (1979) Method and Sarma and Bhave's (1974) Rigorous Method as reported and catalogued by Hoek (1987) are used as base cases for comparison with the results of those generated by Bishop's Method suitably modified for inclusion of internal/external forces/loads.

The Prokon suite of geotechnical computer programs SLOPBG (1994) utilising Bishop's method of analysis and SLOPNC (1994) using the Sarma theory were used for the following evaluations.

INCORPORATING INTERNAL/EXTERNAL LOADS IN BISHOP'S ANALYSIS METHOD

The formulations given in appendix 1 detail the methods adopted in SLOPBG for incorporating internal/external loads in Bishop's analysis method.

SARMA'S ANALYSIS METHOD

The Sarma analysis method was described by Morrison and Dikran (1995:76) as follows:

> The complete multiple wedge method of calculation was discovered by Dr Sarma at Imperial College and first published 15 years ago, Sarma (1979). Development, professional acceptance and commercial use of the method are long overdue. As the method of calculation is easy to understand, numerically simple and less complex than many of the approximate methods, there is no practical reason for its lack of use. The method of calculation has been described in technical literature both by the authors and others, Hoek (1983).
>
> As shown by this review of commercially available slope stability program, none of the user friendly program implement the complete multiple wedge method with reinforcement.

SLOPNC uses Sarma's analysis method. Any shape of failure surface may be analysed and internal and external loads incorporated. No assumptions are made regarding interslice forces and the solution equations satisfy both force and moment equilibrium. The formulations contained in appendix 2 detail the methods as advocated by Sarma and used in SLOPNC.

CORRECTNESS OF METHODS

SLOPBG, utilising BM, usually generates factors of safety that are conservative compared to predictions made using other theories. SLOPNC, utilising Sarma's Method, generates factors of safety that are usually higher but correct.

The BM usually predicts factors of safety that are consistently lower (but not as low as the Ordinary Method of Slices (OMS)) than any of the other commonly used analysis methods. This conservatism is deemed not only to be realistic but also to be advisable where often limited testing is conducted and estimates have usually to be made of shear strength parameters that control stability.

The initial question that must be asked and satisfactorily answered is: 'Which analysis method generates absolutely correct answers?'

Lambe and Whitman (1969:359–362), in a analysis which has remained 'state of the art' even though it was advanced in a text published some 30 years ago, detail a graphical analysis method which generates 'The correct safety factor satisfying statics ...'. This evaluation example, which incorporates a c/Ø slope 20 feet high, subject to a flow net and toe drain, was analysed and shown to exhibit a factor of safety (FOS) of 1.27.

This same slope was input to SLOPNC using Sarma's Method, and analysed to generate an FOS as per figure 1 of 1.266, ie exactly the same as the Lambe and Whitman analysis.

1 A note on nomenclature is necessary. Bishop's 1955 method is termed Bishop's Simplified Method (BSM). Janbu *et al* (1956) further developed the method. The reference manuals for STABR (1985) and STABGM (1985) and SLOPBG (1994) refer to the method as Bishop's Modified Method (BMM) even though the equations used in the formulation are identical to those originally proposed by Bishop. All further reference, either to the BMM or the BSM, have Bishop's original formulations as base and will hereafter be referred to as Bishop's Method (BM).

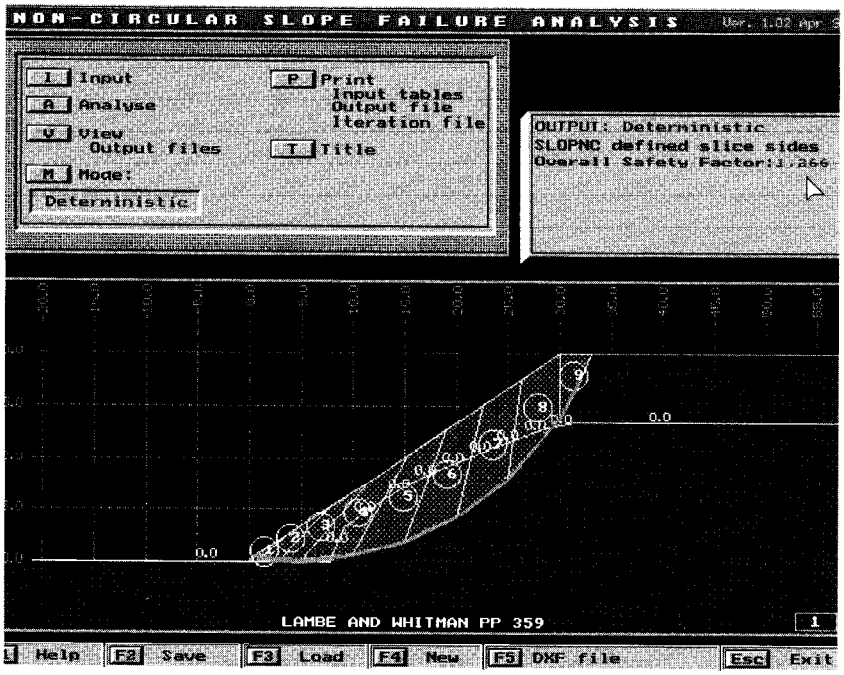

Figure 1 Lambe and Whitman example analysed using SLOPNC

It could be argued that the exact equality generated by this single example may be fortuitous. However, as no assumptions are made in the Sarma analysis method and all conditions of statics are satisfied, and various real-life situations, analysed using the Sarma Method, have indicated it to be correct (Fourie 1994), it may be assumed that the Sarma analysis method used in SLOPNC generates correct answers.

EVALUATION OF VARIOUS METHODS

In comparing various methods, the following facet should be noted. The formulations developed by Bishop contain F, the factor of safety, on both sides of the solution equation. No amount of mathematical manipulation will generate an explicit expression for F in terms of the other variables. Thus an iterative procedure is necessary for solution. The accuracy to which this iteration is conducted usually generates small differences in answers advanced by various researchers. This iteration was difficult in times of manual manipulation, but with the advent of computers is easily accomplished.

In order that comparisons between the BM as used in SLOPBG and the Sarma Method adopted in SLOPNC could be made, initially an example from Fredlund and Krahn (1977) (F+K) was revisited by examining the factors of safety generated by various analysis methods for a particular slope. (Note the original Imperial system of units was retained such that inaccuracies were eliminated.)

Initial evaluation

A theoretical 40 foot high, 1 (vertical) : 2 (horizontal) slope as per figure 2 was analysed.

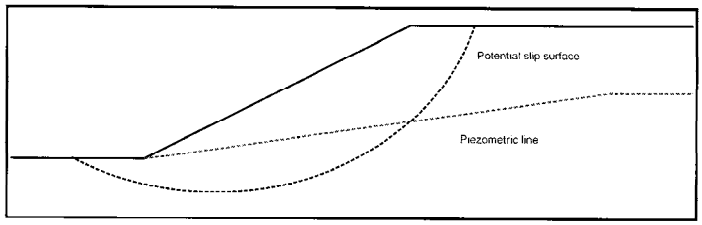

Figure 2 Example from Fredlund and Krahn

The slope was characterised by the following shear and density parameters:

c' = 600 pounds/square foot
Ø' = 20°
y = 120 pounds/cubic foot

Table 1 details the factors of safety (FOS) predicted utilising various analysis methods as reported by F+K.

Table 2 Factors of safety and percentage difference for analysis by SLOPBG and SLOPNC

Method/case	FOS for Bishop's Modified Method (SLOPBG)	Percentage difference in FOS	FOS for Sarma's Method (SLOPNC)
As detailed in figure 2	2.035	*+1.97*	2.076
Figure 2 with ru = 0.25	1.713	*+4.30*	1.790
Figure 2 with piezo-metric line	1.788	*+3.72*	1.857

Because in the Sarma Method

(a) the limit equilibrium model of slope failure is solved reliably
(b) no additional uncertainties are introduced by means of the method of calculation, and
(c) reinforcement and anchor forces are easily incorporated,

the differences between SLOPBG and SLOPNC were further explored by taking the example cited by F+K and

(a) locating a line load of 3 000 pounds/foot at the top of the slope
(b) locating a line load of 600 pounds/foot at mid-height along the slope
(c) using the same configuration as in (b) but with a UDL of 200 pounds/square foot located over the shoulder break point
(d) using the same configuration as in (c) but with a geofabric of 300 pounds/foot strength located at mid-height of the slope

Table 3 details the percentage difference in the calculated FOS between the two methods.

Table 1 Factors of safety for example problem utilising various methods of analysis

Method/ case	Ordinary Method of Slices	Bishop's Simplified Method	Spencer's Method	Janbu's Simplified Method	Janbu's Rigorous Method	Morgenstern-Price Method
As detailed in figure 2	1.928	2.080	2.073	2.041	2.008	2.076
Figure 2 with ru = 0.25	1.607	1.766	1.761	1.735	1.708	1.765
Figure 2 with piezometric line	1.693	1.834	1.830	1.827	1.776	1.833

SLOPBG evaluation

The same theoretical problem as analysed by F+K was re-analysed using SLOPBG and SLOPNC. If Sarma's Method is assumed to generate 'correct answers', factors of safety and the percentages that these factors generated by SLOPBG differ from those as given by SLOPNC are detailed in table 2.

Table 3 Percentage difference in the factors of safety for analysis by SLOPBG and SLOPNC

Method/case	FOS for Bishop's Method (SLOPBG)	Percentage difference	FOS for Sarma's Method (SLOPNC)
Figure 2 with a line load of 3 000 lbs/ft at crest + piezo line	1.750	*+3.74*	1.818
Figure 2 with a line load of 600 lbs/ft along mid-length of slope	2.034	*+1.26*	2.060
Figure 2 with a line load of 600 lbs/ft along mid-length of slope + UDL of 200 lbs/ft over top portions of slope	1.952	*+3.32*	2.019
As above with horizontal geofabric at mid-height	1.956	*+3.22*	2.021

Further evaluations

In order to investigate further the differences between the evaluations using SLOPBG and SLOPNC the example of Lambe and Whitman (1969:359–362) previously detailed was analysed using SLOPBG and SLOPNC with various shear parameters and inclined loads. Table 4 details the results obtained.

Table 4 Percentage difference in the factors of safety for analysis by SLOPBG and SLOPN

Method/case	FOS for Bishop's Method (SLOPBG)	Percentage difference	FOS for Sarma's Method (SLOPNC)
Lambe and Whitman(1969:359–362) with phreatic surface and toe drain using 12 slices: c'= 90 lbs/ft², Ø'= 32°	1.241	+1.97	1.266
Lambe and Whitman(1969:359–362) with phreatic surface and toe drain using 50 slices: c' = 90 lbs/ft², Ø'=32°	1.243	+1.82	1.266
Lambe and Whitman(1969:359–362) with phreatic surface and toe drain using 12 slices: c' = 0 lbs/ft², Ø'=32°	0.939	-2.07	0.920
Lambe and Whitman (1969:359–362) with phreatic surface and toe drain using 12 slices: c' = 0 lbs/ft², Ø'=32°plus an anchor of 1 500 lbs located at mid-slope installed at an angle of 32°	1.018	+0.68	1.025

Figure 3 details the SLOPBG analysis of the same problem analysed using SLOPNC as depicted in figure 1.

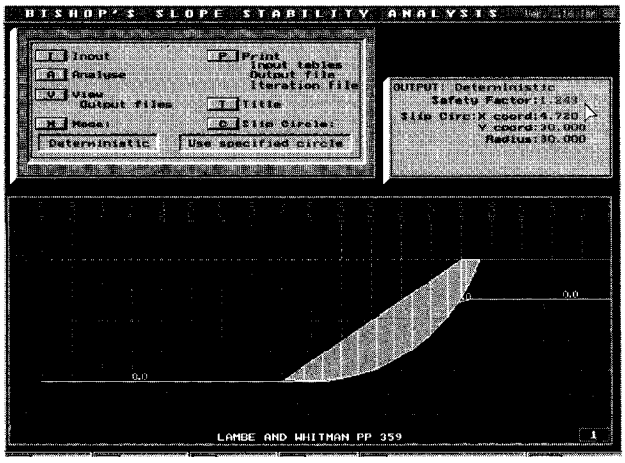

Figure 3 Lambe and Whitman example analysed using SLOPBG

Limitations of SLOPNC

The Sarma method of analysis as utilised in SLOPNC is only complete and satisfies all limiting equilibrium conditions once all tensile stresses on any slice face are eliminated. This is often difficult to accomplish as any small asperities or abrupt discontinuities tend to generate tensile forces. Although SLOPNC utilises, via a method of linear discontinuities, reshaping of constituent slices, it is not always possible to generate a non-negative normal force solution.

CONCLUSION

The limitations as detailed by Heymann (1999) for various internal/external loads in the BM method of slope stability analysis methods are theoretically correct. However, many years of using the BM of analysis for the solution of slope stability problems as incorporated in the computer programs STABR (1985), STABGM (1985) and SLOPBG (1994) have generated confidence that this method predicts realistic lower bound solutions to problems with and without internal/external loads.

The F+K comparisons, although only on a very limited scale, have actually taken one of the worst-case scenarios for the generation of 'correct answers' using BM, as this comparatively deep-seated slip surface usually shows the BM in a very bad light.

Actual differences between BM and the other so-called more correct methods very rarely (if ever) exceed 5%. A difference of 5% is deemed minimal within the context of slope stability factor of safety predictions.

The BM as utilised in SLOPBG generates answers that in the majority of cases are within 5% of the 'true' Sarma SLOPNC method.

SLOPBG is a user-friendly, locally written and inexpensive South African product, which, because of its innovative mouse input method, typically generates answers within ten minutes of start-up. As the above comparisons show, there is thus no reason why it should not be used for slope stability analyses incorporating internal or external loads.

If critical situations are to be analysed, however, where the factor of safety is marginal (1.0 to 1.25) when using SLOPBG, SLOPNC is recommended as the preferred analysis method.

The use of SLOPNC is also recommended where the prospective failure surface is obviously not circular.

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APPENDIX 1: BISHOP'S FORMULATIONS AS USED IN SLOPBG

SLOPBG – Analysis of a generalised soil slope using Bishop's Method

Introduction

SLOPBG is a **slope** stability computer program which uses **B**ishop's Method (1955) of analysis for the evaluation of the stability of generalised soil slopes. The ratio of mobilising and resisting moments on individual slices is used to determine the factor of safety. The slope may be subjected to line and uniformly distributed imposed loads, it may be reinforced with metal strips or geofabric, it may be clad with for example a masonry block wall, and it may be stabilised with anchors. The force profile of the anchor reinforcement may also be input.

The slope may consist of materials with differing shear strength properties, defined either in terms of shear strength parameters c′ and ø′, or an undrained shear strength profile using c on its own. Water pressures are accounted for using either a phreatic line, pore pressure contours or a single pore pressure co-coefficient (r_u). Horizontal and vertical seismic forces are also considered.

The user is presented with a choice of searching for the critical minimum factor of safety (FOS min) circle or he or she may input a user defined circle. Once FOS min is located, the user is given the option of conducting a probabilistic analysis by allowing parameters c′, ø′, γ and the seismic co-efficients a_v and a_h to take on ranges of values within the limits of user defined statistical distributions subject to a user defined range of circle centres. The distribution of the factor of safety subject to the above constraints in the vicinity of FOS min is determined.

Theory

A slope is divided into a number of slices and each slice is analysed as to the forces which are acting on it. Implicit in Bishop's Method (BM) is that the inter-slice forces are horizontal. For the i^{th} slice, the following formulation (from Duncan *et al* 1985) is used.

$$p'l = \frac{W - u(\cos\alpha - c'l\sin\alpha)}{\cos\alpha - (\tan\phi'\sin\alpha)/F} = N' \quad (1)$$

Where N′ = the normal force at the base of the slice, and

$$T = (c'l + p'l\tan\phi)/F \quad (2)$$

Where T′ = the shear strength at the base of the slice
W = weight of the slice
c′ø′ = Mohr-Coulomb strength parameters
u = pore water pressure at base of slice
α = angle of inclination at base of slice
l = length of arc at base of slice
b = width of slic
Ei = side forces

The factor of safety F is given by

$$F = \frac{\Sigma(c'b + (W - ub)\tan\phi'/m_\alpha)}{\Sigma W\sin\alpha} \quad (3)$$

Where

$$m_\alpha = \cos\alpha\ (1 + \tan\phi'\tan\alpha/F) \quad (4)$$

It should be noted that the BM ensures that vertical equilibrium conditions are satisfied, but the formulation suffers from the fact that an iteration procedure is necessary when evaluating the factor of safety using overall moment equilibrium, as from the above equations it can be seen that F occurs on both sides of the equation.

Reinforced slopes

When reinforcement is added to a slope, an additional resisting force is introduced, which coupled with the location of the reinforcement, introduces an additional resisting moment.

Thus if the FOS of a slope is defined as

$$F = \frac{M_R}{M_o} \quad (5)$$

where M_R = resisting moment

and M_o = overturning moment,

then the FOS of a reinforced slope may be expressed as

$$F = \frac{(M_R + M_T)}{M_o} \quad (6)$$

where M_T – resisting moment of the reinforcement

This resisting moment is generated by a force and a lever arm. In the case of a geotextile reinforced slope this force would tend to be tangential to the failure surface whilst for, say, an anchor the force would tend to be in the direction of the anchor provided large movements do not occur.

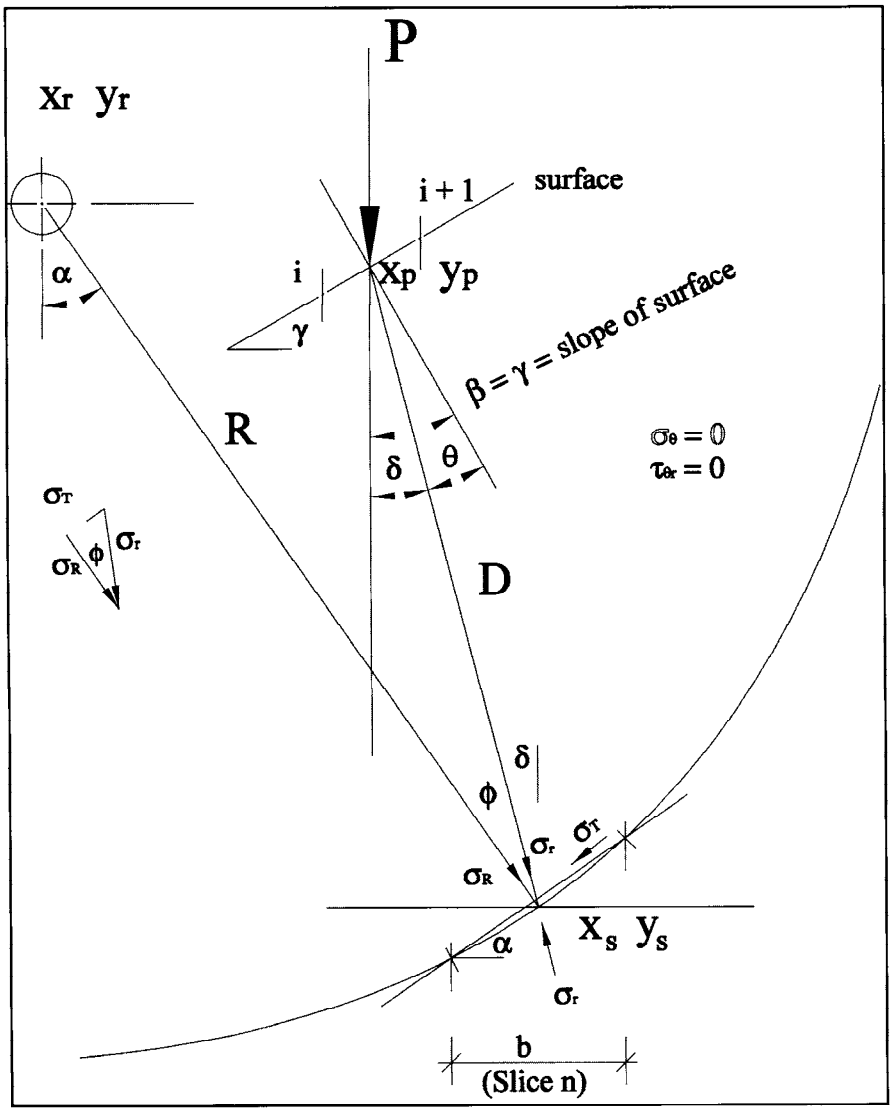
Another factor to bear in mind is the amount of strain necessary to mobilise the force. In the case of a geotextile this may be significant (say 10%). The designer should note this fact and scale down the ultimate strength of the reinforcement to reflect the amount of movement anticipated to mobilise this force.

External loadings

The following formulations adapted from Harr (1966) have been used.

Line loads

From the figure below



$$D = \sqrt{(x_s - x_p)^2 + (y_p - y_s)^2} \quad (7)$$

$$\delta = \sin^{-1}\left(\frac{x_s - x_p}{D}\right) \quad (8)$$

$$\theta = \beta - \delta \quad (9)$$

$$\sigma_r = \frac{2P\cos\delta}{\pi D} \quad (10)$$

$$\phi = \alpha - \delta \quad (11)$$

$$\sigma_T = \sigma_r \sin\phi \quad (12)$$

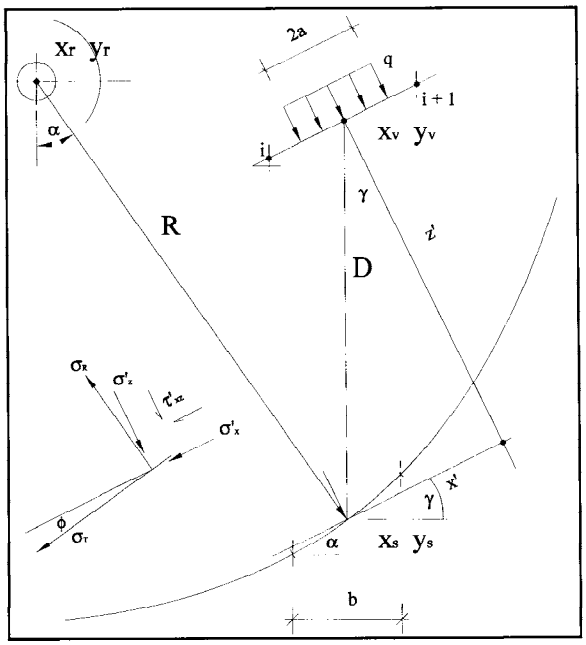
The term σR acts in the direction from the centre of the slip circle in question to the centre of the slice under consideration, and thus does not provide any additional mobilizing or resisting moment. The line load acting at a surface thus only provides an additional mobilising force on the slice in question equal to

$$\frac{\sigma_T R b}{\cos\alpha} \quad (13)$$

Uniformly distributed load

Two cases will be considered; normal to any surface, and tangential to the surface.

(a) Normal component



Using the same notations as previously, ie as per the sketch above, we have for a UDL of width 2a, intensity w, inclined at an angle *y* to the surface, that

$$q = w\cos\gamma \quad (14)$$

The following geometric considerations may be derived

$$D = \sqrt{(x_s - x_v)^2 + (y_s - y_v)^2} \quad (15)$$

$$z' = D\cos\gamma \quad (16)$$

$$x' = D\sin\gamma \quad (17)$$

where

$$\gamma = \tan^{-1}\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right) \quad (18)$$

From the formulations of Harr (1966) the following expressions may be derived for the normal, tangential and shear stresses:-

$$\sigma_{z'} = \frac{q}{\pi}\left[\tan^{-1}\left(\frac{z'}{x'-a}\right) - \tan^{-1}\left(\frac{z'}{x'+a}\right) - \frac{2az'(x'^2 - z'^2 - a^2)}{(x'^2 + z'^2 - a^2)^2 + 4a^2z'^2}\right] \quad (19)$$

$$\sigma_{x'} = \frac{q}{\pi}\left[\tan^{-1}\left(\frac{z'}{x'-a}\right) - \tan^{-1}\left(\frac{z'}{x'+a}\right) + \frac{2az'(x'^2 - z'^2 - a^2)}{(x'^2 + z'^2 - a^2)^2 + 4a^2z'^2}\right] \quad (20)$$

$$\tau'_{xz} = \frac{4aqx'z'^2}{\pi((x'^2 + z'^2 - a^2)^2 + 4a^2z'^2)} \quad (21)$$

to transform the co-ordinate system.

$$\phi = \alpha - \gamma \quad (22)$$

Now using the Mohr-Coulomb failure condition circles we have

$$\sigma_T = \frac{\sigma_{x'} + \sigma_{z'}}{2} + \frac{\sigma_{x'} - \sigma_{z'}}{2}\cos 2\phi + \tau'_{xz}\sin 2\phi \quad (23)$$

The normal component of the UDL thus generates a mobilising force of

$$\frac{\sigma_T R b}{\cos\alpha} \quad (24)$$

which acts on the base of the slice under consideration.

(b) Tangential component

In this case

$$q = w\sin\alpha \quad (25)$$

As previously in equations 15 to 18 we have

$$D = \sqrt{(x_s - x_v)^2 + (y_p - y_v)^2} \quad (26)$$

$$z' = D\cos\gamma \quad (27)$$

$$x' = D\sin\gamma \quad (28)$$

Where

$$\gamma = \tan^{-1}\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right) \quad (29)$$

Now from Harr (1966:58-62) for the normal, tangential and shear stresses we have

$$\sigma_z' = \frac{4aqx'z'^2}{\pi(x'^2 + z'^2 - a^2)^2 + 4a^2z'^2} \quad (30)$$

$$\sigma_x' = \frac{q}{\pi}\left[\ln\left(\frac{(x'+a)^2+z'^2}{(x'-a)^2+z'^2}\right) - \frac{4ax'z'^2}{(x'^2+z'^2-a^2)^2+4a^2z'^2}\right] \quad (31)$$

$$\tau'_{xz} = \frac{q}{\pi}\left[\tan^{-1}\left(\frac{z'}{x'-a}\right) - \tan^{-1}\left(\frac{z'}{x'+a}\right) - \frac{2az'(x'^2-z'^2-a^2)}{(x'^2+z'^2-a^2)^2+4a^2z'^2}\right] \quad (32)$$

and as previously to transform the co-ordinate system

$$\phi = \alpha - \gamma \quad (33)$$

Now using the Mohr-Coulomb failure condition circles we have

$$\sigma_T = \frac{\sigma_x' + \sigma_z'}{2} + \frac{\sigma_x' - \sigma_z'}{2}\cos 2\phi + \tau'_{xz}\sin 2\phi \quad (34)$$

The tangential component of the UDL thus generates a mobilising force of

$$\frac{\sigma_T R b}{\cos\alpha} \quad (35)$$

which acts at the base of the slice under consideration.

Notes:
- The above formulations have been undertaken to model the effects of loads on the stability of the system. It is realised that they probably violate the conditions imposed for inter-slice forces. However, the effects of this violation are probably small and will not alter materially the fact that the BM is in itself only an approximation for the actual situation.
- The imposed loads are only deemed to have an effect if they fall inside that portion of the soil mass which has the potential for failure.

Seismic loading

Seismic events induce loads on the structure which act against stability. Both vertical and horizontal seismic forces are assumed to act through the centre of mass of each slice.

Tension cracks

In soils which exhibit a significant cohesion (c') intercept it is possible that the soil may stand unsupported to some height. This height is the depth to which a tension crack may form in the slope.

The formulation for the depth of the tension crack is given by Lambe and Whitman (1969:342) as

$$Z_c = \frac{2c'\sqrt{N_\phi}}{\gamma} \quad (36)$$

where c' = the cohesion intercept in that portion of the embankment

$$N_\phi = \frac{1 + Sin\,\phi'}{1 - Sin\,\phi'} \quad (37)$$

and γ = The unit weight of the material

Water in tension cracks

Water pressure is always assumed to act at right angles to the failure surface.

APPENDIX 2: SARMA'S FORMULATIONS AS USED IN SLOPNC

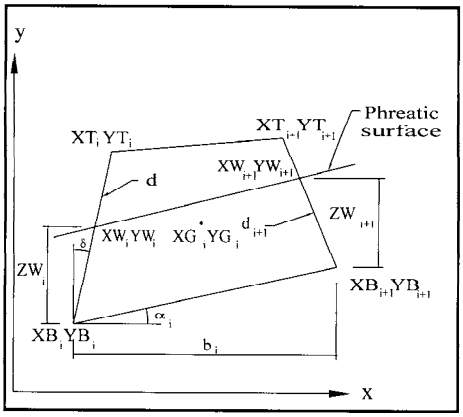
SLOPNC – Non-circular slope failure

Introduction

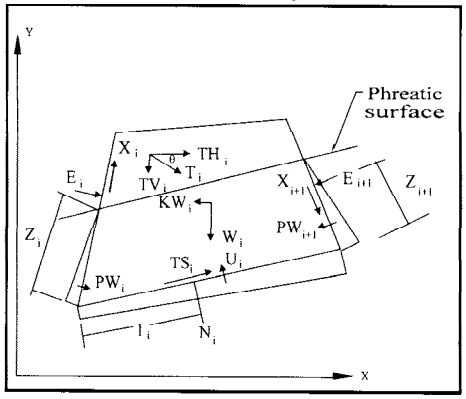
SLOPNC is a generalised non-circular slope stability program which uses the non-vertical slice method as proposed by Sarma (1979) for the prediction of the factor of safety of general shape surfaces. As the boundaries are non-vertical, structural features such as faults or discontinuity planes may be included. Water pressures, external loadings and reinforcement have been included so as to make the analysis as generalised as possible.

Theory

Consider a generalised quadrilateral within a soil or rock mass as depicted in the figure below:



The following forces as per Hoek (1983) may act on the soil or rock segment.



The following geometric considerations as per Hoek (1983) may be derived:

$$d_{i+1} = ((XT_{i+1} - XB_{i+1})^2 + (YT_{i+1} - YB_{i+1})^2)^{1/2} \quad (1)$$

$$\delta_{i+1} = Arcsin((XT_{i+1} - XB_{i+1})/d_{i+1} \quad (2)$$

$$b_i = XB_{i+1} - XB_i \quad (3)$$

$$\alpha_i = Arctan((YB_{i+1} - YB_i)/b_i \quad (4)$$

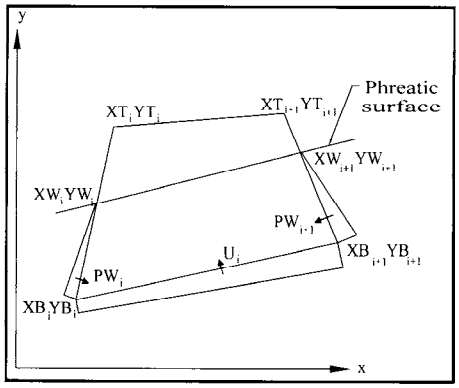
$$W_i = 1/2\gamma((YB_i - YT_{i+1})(XT_i - XB_{i+1}) + (YT_i - YB_{i+1})(XT_{i+1} - XB_i)) \quad (5)$$

$$ZW_{i+1} = (YW_{i+1} - YB_{i+1}) \quad (6)$$

The submergence condition dictates the forces acting on the sides of the quadrilateral. The uplift water force acting on the base of the slice is in all cases given by

$$U_i = 1/2\gamma_W(YW_i - YB_i + YW_{i+1} - YB_{i+1})b_i\,Sec\alpha_i \quad (7)$$

Case 1: No submergence of slice, as detailed in the figure below:

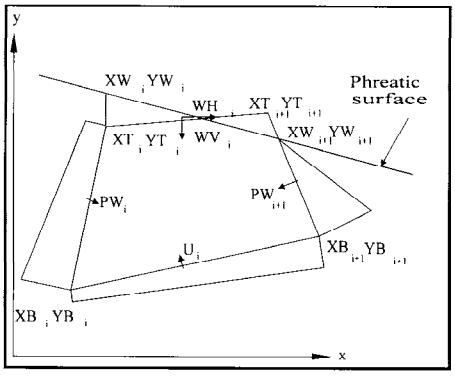


The side forces are evaluated as

$$PW_i = 1/2\gamma_W(YW_i - YB_i)^2\,Sec\delta_i \quad (8)$$

$$PW_{i+1} = 1/2\gamma_W(YW_{i+1} - YB_{i+1})^2\,Sec\delta_i \quad (9)$$

Case 2: Submergence of side i only, as detailed in the figure below:



$$YT_i < YW_i \qquad YT_{i+1} > YW_{i+1} \quad (10)$$

The side forces are evaluated as

$$PW_i = 1/2\gamma_W((2YW_i - YT_i - YB_i)(YT_i - YB_i)/Cos\delta_i) \quad (11)$$

$$PW_{i+1} = 1/2\gamma_W(YW_{i+1} - YB_{i+1})^2/Cos\delta_{i+1}) \quad (12)$$

The vertical and horizontal forces applied to surface of the slice as a result of sub-

mergence of part of the slice are given by

$$WW_i = 1/2\gamma_W (YW_i - YT_i)^2 (XT_{i+1} - XT_i)/(YT_{i+1} - YT_i)) \quad (13)$$

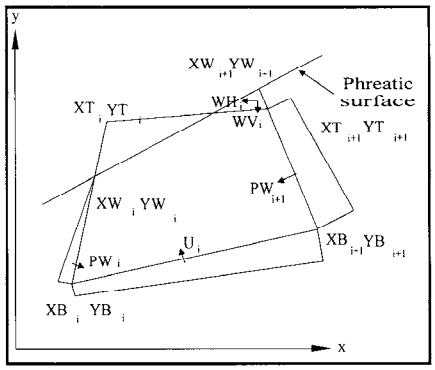
$$WH_i = 1/2\gamma_W (YW_i - YT_i)^2 \quad (14)$$

Case 3: Submergence of side i+1 only, as detailed in the figure below:

$$YT_i > YW_i \qquad YT_{i+1} < YW_{i+1} \quad (15)$$

The side forces are evaluated as

$$PW_i = 1/2\gamma_W ((YW_i - YB_i)^2/Cos\delta_i) \quad (16)$$

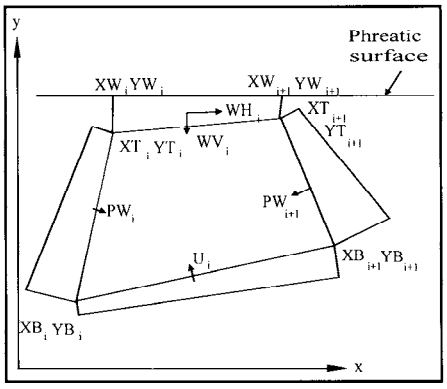


The vertical and horizontal forces applied to surface of the slice as a result of submergence of part of the slice are given by

$$WW_i = 1/2\gamma_W (YW_{i+1} - YT_{i+1})^2 (XT_{i+1} - XT_i)/(YT_{i+1} - YT_i)) \quad (18)$$

$$WH_i = 1/2\gamma_W (YW_{i+1} - YT_{i+1})^2 \quad (19)$$

Case 4: Complete submergence of slice i, as detailed in the figure below:



$$XT_i < XW_i \qquad XT_{i+1} < XW_{i+1} \quad (20)$$

The side forces are evaluated as

$$PW_i = 1/2\gamma_W ((2YW_i - YT_i - YB_i)(YT_i - YB_i)/Cos\delta_i) \quad (21)$$

$$PW_{i+1} = 1/2\gamma_W (2YW_{i+1} - YT_{i+1} - YB_{i+1})(YT_{i+1} - YB_{i+1})/Cos\delta_{i+1}) \quad (22)$$

The vertical and horizontal forces applied to surface of the slice as a result of submergence of part of the slice are given by

$$WW_i = 1/2\gamma_W (YW_i - YT_i - YW_{i+1} - YT_{i+1})(XT_{i+1} - XT_i)) \quad (23)$$

Water forces on the first and last slices

Although the water force PWi acting on the first slice side and the force PW_{n+1} acting on the $(n+1)^{th}$ slice side (which could be a tension crack) are calculated by means of the equations listed above, these forces are not normally used in the calculation of the critical acceleration. The simplest way to incorporate these forces into the analysis is to treat them as external forces with the following components

$$TV_i = PW_i Sin\delta_i \quad (25)$$

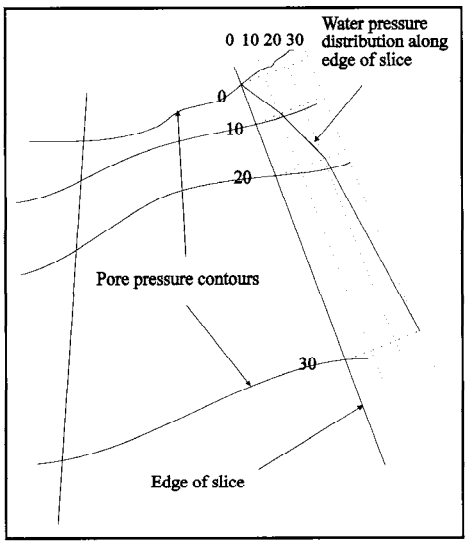
$$TH_i = PW_i Cos\delta_i \quad (26)$$

$$TV_n = PW_{n+1} Sin\,\delta_{n+1} \quad (27)$$

$$TH_n = PW_{n+1} Cos\,\delta_{n+1} \quad (28)$$

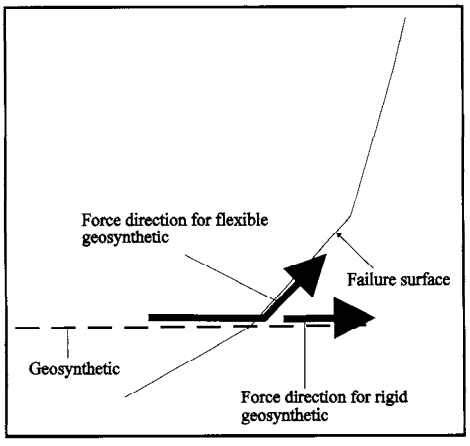
Pore pressure contours

In the case where pore pressure contours are used to characterise the moisture condition, the resultant nett load distribution due to the water regime is applied to the side of the slice in question. The figure below details the approach adopted.



Geosynthetic loadings

The presence of a geosynthetic (gs) at a slice boundary is handled as follows. If the gs is flexible, then the resistance generated by the gs will tend to be in direction of the shear boundary along which movement, prior to failure, would take place. If on the other hand the gs is rigid, ie a very stiff gs or a soil nail, then the tensile force within the gs would act in the direction of the member, as illustrated in the sketch below. The user is provided with this choice in the program.



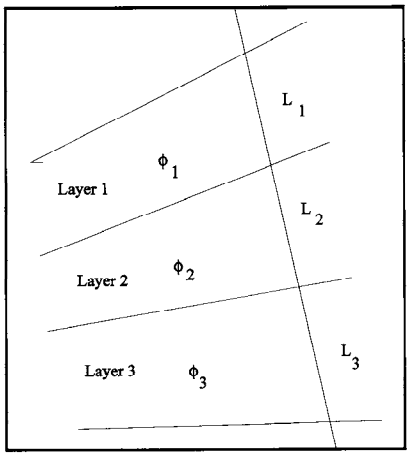
Other external loadings

These are assumed to act at the positions which they physically occupy on the slice in question. These loads may for example be line loads or UDL's or they may be the position at which an anchor is applied. Their effect is evaluated by resolving into vertical and horizontal components.

Multiple layers

The program possesses the capability of handling multiple layers within one slice. For the evaluation of the Qi parameter in the critical acceleration equations as presented hereafter, we have used the following formulation for the average frictional component over the slice in question, based on the figure given below:

$$\bar{\phi} = Cos^{-1}\left(\frac{\sum_{i=1}^{n} Cos\phi_i L_i}{\sum_{i=1}^{n} L_i}\right) \quad (29)$$



Calculation of the critical acceleration

The critical acceleration required to bring the slope to a condition of limiting equilibrium is given by

$$K_c = \frac{a_n + a_{n-1}e_n + a_{n-2}e_n e_{n-1} + \ldots a_1 e_n e_{n-1} \ldots e_3 e_2}{p_n + p_{n-1}e_n + p_{n-2}e_n e_{n-1} + \ldots p_1 e_n e_{n-1} \ldots e_3 e_2} \quad (30)$$

$$+S_{i+1}Sin(\phi'_{Bi} - \alpha_i - \delta_{i+1}) - S_i\,Sin(\phi'_{Bi} - \alpha_i - \delta_i)) \quad (31)$$

$$+S_{i+1}Sin(\phi'_{Bi} - \alpha_i - \delta_{i+1}) - S_i\,Sin(\phi'_{Bi} - \alpha_i - \delta_i)) \quad (32)$$

The factor of safety is calculated by reducing the shear strength parameters simultaneously on all sliding surfaces until the resultant acceleration at the centre of mass of the slice is

$$p_1 = Q_i W_i Cos(\phi'_{Bi} - \alpha_i) \quad (33)$$

$$e_i = Q_i(Cos(\phi'_{Bi} - \alpha_i + \phi'_{Si} - \delta_i)/Cos\phi'_{si}) \quad (34)$$

$$Q_i = Cos\phi'_{Si}/Cos(\phi'_{Bi} - \alpha_i + \phi'_{Si+1} - \delta_{i+1}) \quad (35)$$

$$S_i = c'_{Si}d_i - PW_i tan\phi'_{Si} \quad (36)$$

$$S_{i+1} = c'_{Si+1} d_{i+1} - PW_{i+1} \tan\phi'_{Si+1} \quad (37)$$

$$R_i = c'_{Bi} b_i / Cos\alpha_i - U_i \tan\phi'_{Bi} \quad (38)$$

equal to the resultant acceleration specified. This is done by progressively reducing the shear strength parameters in the equations above by

$$c'_{Bi} = c_{Bi} / F \quad (39)$$

$$c'_{Si} = c_{Si} / F \quad (40)$$

$$c'_{Si+1} = c_{Si+1} / F \quad (41)$$

$$\phi'_{Bi} = Tan^{-1} (\tan \phi_{Bi} / F) \quad (42)$$

$$\phi'_{Si} = Tan^{-1}(\tan \phi_{Si} / F) \quad (43)$$

$$\phi'_{Si+1} = Tan^{-1}(\tan \phi_{Si+1} / F) \quad (44)$$

When the value of K for a given factor of safety has been found, the forces acting on the sides and base of each slice are found by progressive solution of the following equations, starting from the known condition that $E_1 = 0$.

$$E_{i+1} = a_i - p_i K + E_i e_i \quad (45)$$

$$X_i = (E_i - PW_i) Tan\phi'_{Si} + c'_{Si} d_i \quad (46)$$

$$N_i = (W_i + TV_i - X_{i+1} Cos\delta_{i+1} + X_i Cos\delta_i - E_{i+1} Sin\delta_{i+1} + E_i Sin\delta_i + U_i Tan\phi'_{Bi} Sin\alpha_i - c'_{Bi} b_i Tan\alpha_i) Cos\phi'_{Bi} / Cos(\phi'_{Bi} - \alpha_i) \quad (47)$$

The effective normal stresses acting across the base and sides of the slice are calculated thus:

$$TS_i = (N_i - U_i) Tan\phi'_{Bi} + c'_{Bi} b_i / Cos\alpha_i \quad (48)$$

$$\sigma_{Bi} = (N - U_i) Cos\alpha_i / b_i \quad (49)$$

$$\sigma_{Si} = (E - PW_i) / d_i \quad (50)$$

$$\sigma_{Si+1} = (E_{i+1} - PW_{i+1}) / d_{i+1} \quad (51)$$

In the above equations, the effective normal stresses must be positive for the solution to be valid.

As a final check, Sarma (1979) recommends that moment equilibrium conditions must be satisfied for each slice. Referring to the figure at the start of this addendum and taking moments about the lower left hand corner, we have:

$$N_i / -X_{i+1} b_i Cos(\alpha_i + \delta_{i+1}) / Cos\alpha_i - E_i Z_i \quad (52)$$

$$+ E_{i+1} [Z_{i+1} + b_i Sin(\alpha_i + \delta_{i+1}) / Cos\alpha_i] \quad (53)$$

$$- W_i (XG_i - X_{Bi}) + K_c W_i (YG_i - Y_{Bi}) - TV_i (X_i - XG_i) \quad (54)$$

$$+ TH_i (Y_i - YG_i) = 0 \quad (55)$$

Starting from the slice at the toe of the slope, where $Z_i = 0$, assuming a value of 1_i, the moment arm Z_{i+1} can be calculated or vice versa. The values of Z_i and Z_{i+1} should lie within the slice boundary, preferably in the middle third.