A refined approach to lateral-torsional buckling of overhang beams

Economic benefit of ensuring uninterrupted water supply during prolonged electricity disruptions – City of Tshwane case study

Comparing the factors of safety from finite element and limit equilibrium analyses in lateral support design

Small-scale dispersed Green Infrastructure – a fitting civil engineering solution to stormwater quality improvement?

Concrete properties for ultra-thin continuously reinforced concrete pavements (UTCRCP)

The development of suitable cyclic loading and boundary conditions for ballast box tests

Mechanical properties, durability characteristics and shrinkage of plain cement and fly ash concretes subjected to accelerated carbonation curing
GUIDELINES FOR THE PREPARATION OF PAPERS AND TECHNICAL NOTES

Authors should comply with the following guidelines when preparing papers for publication in the Journal of the South African Institution of Civil Engineering. The Journal is published quarterly in March, June, September and December. Articles submitted for publication are reviewed by a panel of referees under the guidance of the SAICE Journal Editorial Panel. The journal publishes research papers covering all the disciplines of civil engineering (structural, geotechnical, railway, coastal/marine, water, construction, environmental, municipal, transportation) and associated topics that are relevant to the civil engineering profession, and that preferably have some relevance to civil engineering in southern Africa and the African continent.

When preparing articles for publication, authors should please take note of the following and comply with the guidelines as set out:

CLASSIFICATION OF ARTICLES CONSIDERED FOR PUBLICATION

1. Technical papers are well-researched, in-depth, fully-referenced technical articles not exceeding 6 000 words in length (excluding tables, illustrations and the list of references). Related papers that deal with “softer sciences” (e.g., education, social upliftment, etc.) are accepted if they are of a technical nature and of particular interest to the civil engineering profession. The latter type of paper will be subject not only to peer-review by civil engineers, but also to review by non-engineering specialists in the field covered by the paper.

2. Technical notes are short, fully-referenced technical articles that do not exceed 2 000 words. A typical technical note will have limited scope often dealing with a single technical issue of particular importance to civil engineering.

3. Review papers are considered for publication as either technical papers or technical notes on conditions that they are the original work of the author and will assist the reader with the understanding, interpreting or applying of the subject under review. A review paper must contain criteria by which the work under review was evaluated, and contribute by synthesising the information and drawing new conclusions from the dissemination of the previously published work.

4. Discussion on published articles is welcomed up to six months after publication. The length of discussion contributions is limited to 1 500 words. Where appropriate, discussion contributions will be subject to the normal reviewing process and will be forwarded to the authors of the original article for reply.

POLICY REGARDING LANGUAGE AND ORIGINALITY OF SUBMITTED ARTICLES

1. Language: Manuscripts should preferably be presented in English, as the journal is distributed internationally. Articles submitted in any of the other official South African languages should be accompanied by an expanded abstract in English.

2. Original work: Papers and technical notes must be original contributions. Authors must affirm that submitted material has not been published previously, is not under consideration for publication elsewhere and will not be submitted elsewhere while under consideration by the SAICE Journal Editorial Panel. It is the responsibility of the authors to ensure that publication of any paper in the journal will not constitute a breach of any agreement or the transgression of any law. The corresponding author should confirm that all co-authors have read and approved the manuscript and accept these conditions. Authors are responsible for obtaining permission to publish experimental data and other information that may be confidential or sensitive. Authors are also responsible for obtaining permission from copyright owners when reproducing material that has been published elsewhere. Proof of such permission must be supplied.

SUBMISSION PROCEDURES AND REQUIRED FORMAT

1. Online submission: Manuscripts must be uploaded as PDF files (http://journal.saice.org.za). Individual file sizes may not exceed 10 MB. Should you experience problems uploading your paper, please contact the editor (verelen@saice.org.za).

2. Format: Manuscripts should be prepared in MS Word and presented in double line spacing, single column layout with 25 mm wide margins. Line numbers must be applied to the whole document. All pages should bear the authors’ names and be numbered at the bottom of the page. With the exception of tables and figures (see below) the document should be typed in Times New Roman 12 pt font. Contributions should be accompanied by an abstract of not more than 200 words.

3. First page: The first page of the manuscript should include the title of the paper, the number of words of the main text (i.e., excluding figures, tables and the list of references), the initials and surnames of the authors, professional status (if applicable), SAICE affiliation (Member, Fellow, Visitor, etc), telephone numbers (landline and mobile), and e-mail and postal addresses. The name of the corresponding author should be underlined. Five keywords should be suggested.

4. Figures, tables, photos and illustrations: These should preferably be submitted in colour, as the journal is a full-colour publication.

5. Their positions should be clearly marked in the text as follows: [Insert Figure 1].

6. Figures, tables, photos, illustrations and equations should be numbered consecutively and should appear in the text directly after they have been referred to for the first time.

7. Illustrations must be accompanied by appropriate captions. Captions for tables should appear above the table. All other captions should appear below the illustration (figures, graphs, photos).

8. Only those figures and photographs essential to the understanding of the text should be included. All illustrations should be referred to in the text. Figures should be produced using computer graphics. Hand-drafted figures will not be accepted. Lettering on figures should be equivalent to a Times New Roman 9 pt font or slightly larger (up to 12 pt) if desired. Lettering smaller than 9 pt is not acceptable.

9. Tables should be typed in Times New Roman 9 pt font. They should not duplicate information already given in the text, nor contain material that would be better presented graphically. Tabular matter should be as simple as possible, with brief column headings and a minimum number of columns.

10. Mathematical expressions and presentation of symbols:

   a. Equations should be presented in a clear form which can easily be read by non-mathematicians. Each equation should appear on a separate line and should be numbered consecutively.

   b. Symbols should preferably reflect those used in Microsoft Word Equation Editor or Mathtype, or should be typed using the Times New Roman symbol set.

   c. Variables in equations (x, y, z, etc. as well as lower case Greek letters) should be presented in italics. Numbers (digits), upper case Greek letters, symbols of metric measurement units (m for metres, s for seconds, etc) and mathemati-cal/ trigonometrical functions (such as sin, cos and tan) are not written in italics, but in upright type (Roman). Variables and symbols used in the body of the text should match the format used in the equations, i.e. upright or italics, whichever is applicable.

   d. Metric measurement abbreviations/units should conform to international usage – the SI system of units should be used.

   e. Decimal commas may be used, but decimal points are preferred.

   f. Symbols should preferably be defined in the text, but if this is not feasible, a list of notations may be provided for inclusion at the end of the paper.

11. Headings: Sections and paragraphs should not be numbered. The following hierarchy of headings should be followed:

   HEADING OF MAIN SECTION

   Heading of sub-section

   Heading of sub-sub-section

12. References: References should follow the Harvard system. The format of text citations should be as follows: “Jones (1999) discovered that…” or “recent results (Brown & Carter 1985; Grean et al 1999) indicated that…”

   a. References cited in the text should be listed in alphabetical order at the end of the paper. References by the same author should be in chronological order.

   b. The following are examples of a journal article, a book and a conference paper:


13. Papers published previously in the Journal of the South African Institution of Civil Engineering should be cited if applicable.

14. Footnotes, trade names, acronyms, abbreviations: These should be avoided. If acronyms are used, they should be defined when they first appear in the text. Do not use full stops after abbreviations or acronyms.

15. Return of amended papers: Papers requiring amendments will be accepted up to six months after the referee reports had been sent to authors, after which the paper will be withdrawn from the system.

FINAL ARTICLE

1. Copyright: On acceptance of the paper or technical note, copyright must be transferred by the author/s to the South African Institution of Civil Engineering on the form that will be provided by the Institution.

2. Photos of authors: The final corrected version of the paper should be accompanied by recent, high-resolution head and shoulders colour photographs and a profile not exceeding 100 words for each of the authors.

3. Proofs: First proofs of papers will be sent to authors in PDF format for verification before publication. No major re-writes will be allowed, only essential minor corrections.

The Journal of the South African Institution of Civil Engineering is published quarterly in March, June, September and December. Articles submitted for publication are reviewed by a panel of referees under the guidance of the SAICE Journal Editorial Panel. The journal publishes research papers covering all the disciplines of civil engineering (structural, geotechnical, railway, coastal/marine, water, construction, environmental, municipal, transportation) and associated topics that are relevant to the civil engineering profession, and that preferably have some relevance to civil engineering in southern Africa and the African continent.
A refined approach to lateral-torsional buckling of overhang beams
S H Venter, S A Skorpen, B W J van Rensburg
DOI: 10.17159/2309-8775/2019/v61n4a1

Economic benefit of ensuring uninterrupted water supply during prolonged electricity disruptions – City of Tshwane case study
J C Potgieter, C Herold, M van Dijk, J N Bhagwan
DOI: 10.17159/2309-8775/2019/v61n4a2

Comparing the factors of safety from finite element and limit equilibrium analyses in lateral support design
J-T Potgieter, S W Jacobsz
DOI: 10.17159/2309-8775/2019/v61n4a3

Small-scale dispersed Green Infrastructure – a fitting civil engineering solution to stormwater quality improvement?
I C Brink
DOI: 10.17159/2309-8775/2019/v61n4a4

Concrete properties for ultra-thin continuously reinforced concrete pavements (UTCRCP)
A Mackellar, E Kearsley
DOI: 10.17159/2309-8775/2019/v61n4a5

The development of suitable cyclic loading and boundary conditions for ballast box tests
T C U Jideani, P J Gräbe
DOI: 10.17159/2309-8775/2019/v61n4a6

Mechanical properties, durability characteristics and shrinkage of plain cement and fly ash concretes subjected to accelerated carbonation curing
R A Assaggaf, S K Adekunle, S Ahmad, M Maslehuiddin, O S B Al-Amoudi, S I Ali
DOI: 10.17159/2309-8775/2019/v61n4a7
A refined approach to lateral-torsional buckling of overhang beams

S H Venter, S A Skorpen, B W J van Rensburg

The current South African Steel design code, SANS 10162-1, has a set of effective length factors for overhang beams which is independent of the geometrical properties of the beam and the lengths of the backspan and cantilever. This simple approach is consistent with several other international steel design codes and design guidelines. These effective length factors make no allowance for the stiffness of the adjacent span, but in reality warping at the supports allows interaction buckling between the cantilever and beam segments.

In the research presented in this paper the backspan-to-overhang-segment ratio was investigated with the view of refining the calculations for determining the critical buckling moment of overhang beams. The scope was limited to beams with lateral and torsional restraints at the supports, and to shear centre and top flange loading applied at the free overhang end. Physical experiments and finite solid element analyses were used to determine the relationship between the critical moments and the beam buckling parameters. A simplified design calculation procedure was formulated, which includes a buckling parameter to include warping at the supports and allows interaction buckling between the beam segments. The buckling parameter is dependent on the size of the beam, the length of the overhanging segment and the ratio of backspan-to-overhang length.

INTRODUCTION

All structural steel design codes allow for the design of beams that are susceptible to buckling. A possible mode of buckling for slender beams is lateral-torsional buckling (LTB). An elastic critical moment ($M_{cr}$) is determined, where $M_{cr}$ dictates the resistance of slender beams. The plastic moment of resistance ($M_{pl}$) limits the capacity of stocky beams. Transitional equations predict the resistance between the extremes $M_{pl}$ and $M_{cr}$. In the transitional zone out-of-straightness and residual stresses play a significant role. SANS 10162-1 (2011) provides effective length factors which take the effect of support loading conditions into account. The effective length factors for cantilevers are simplified numbers and they do not take the torsional properties parameter or the backspan-to-overhang-length ratio into account.

SANS 10162-1 (2011) uses effective length factors for cantilevers adapted from Ziemian (2010), whose work is based on the original research of Kirby and Nethercot (1979). Kirby and Nethercot (1979) specified that the effective length factors were limited to beams with overhang effective lengths greater than or equal to the backspan effective length. This limit was subsequently omitted by Ziemian (2010) and is also not stipulated in SANS 10162-1 (2011). However, the LTB capacity of a beam is dependent on the magnitude of warping of the entire beam, which is influenced by adjacent spans.

The purpose of the study was to investigate the effect that the backspan has on the LTB capacity of a bi-symmetrical overhang I-beam. The scope of the study was limited to overhang supports restraining lateral and torsional movement, and the application of load was limited to a concentrated point force at the free end of the overhang beam applied to the shear centre or to the top flange. Two methods were used to determine the buckling capacity of overhang beams, namely physical experiments and finite element modelling (FEM). The physical experiments were limited to an I-beam, the IPE_{AA} 100. (The geometrical properties of this I-section are given in Table 10.) The physical experiments served as the control to which the solid element FEM analyses were calibrated and expanded. A parametric study using FEM was then conducted with the aim of assessing the effect of beam size, overhang length, load height and...
backspan-to-overhang-length ratio on the critical buckling moment.

The results of this research point to a possible refined approach to the design of overhang beams, which includes the effect of an adjacent span on the LTB behaviour of an overhang beam.

OVERHANG STEEL BEAMS

Overhang beams are continuous beams where the end span is cantilevered. The main difference between the cantilevered segment of a built-in cantilever and overhang beam is the warping restraint at the support. With a built-in cantilever, warping is prevented, whereas in an overhang beam, not only is warping allowed, but warping also depends on the relative stiffness of the adjacent span. The LTB stiffness of the adjacent span depends on the size of the beam, the laterally unbraced length, and the loading on that segment.

Timoshenko and Gere (1961) proved that it was possible to formulate a closed-form solution for elastic lateral-torsional buckling for both a simply supported beam and a built-in cantilever. They considered an unbraced built-in cantilever with a point load at the tip of the cantilever at the shear centre. The solution depends on the length, the torsional stiffness and warping rigidity of the beam. Elastic LTB refers to buckling that occurs without permanent deformation and depends on the lateral slenderness of the beam. With elastic LTB the yield strength and residual stresses are not considered. In addition, with this model the interaction with local buckling or distortional buckling is not considered.

Kirby and Nethercot (1979) introduced an effective length factor to account for the various support and loading conditions possible in cantilevers and overhang beams. Currently SANS 10162-1 (SANS 2011) uses Equation 1 to determine the LTB capacity of a beam. This formula is based on Timoshenko and Gere’s (1961) simply supported beam equation, but modified to incorporate an effective length factor. The effective length factors depend on the restraint conditions for rotation about the minor axis and the warping restraint at the supports, and also on the destabilising or normal load conditions. No provision is made for the effect of the lateral buckling length of an adjacent span or the torsional parameter (defined later) on the effective length factor. Even though Equation 1 is based on a simply supported beam, SANS 10162-1 (SANS 2011; also BS 2008 and Ziemian 2010) uses this equation in conjunction with an effective length factor for cantilevers and overhang beams ($\omega_2 = 1$ in the case of a cantilever with no effective lateral support for the beam at the free end).

$$M_{cr} = \frac{\pi}{kL} \sqrt{\frac{EJy}{kl}} \left[ \frac{kE^2}{kL} \right] \left[ \frac{Iy}{Iw} \right] (1)$$

Where:
- $M_{cr}$ = elastic critical moment of a beam segment
- $k$ = effective length factor (in SANS 10162-1)
- $L$ = length of beam between lateral restraints, projecting length of cantilever
- $E$ = elastic modulus of steel
- $J_y$ = moment of inertia about y-axis (minor axis)
- $G$ = shear modulus of steel
- $J$ = St Venant torsion constant of a cross-section
- $C_w$ = warping torsional constant.

LIST OF NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Design equation factor</td>
</tr>
<tr>
<td>B</td>
<td>Design equation factor</td>
</tr>
<tr>
<td>C</td>
<td>Torsional rigidity (Timoshenko &amp; Gere 1961), design equation factor</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Warping rigidity (Timoshenko &amp; Gere 1961)</td>
</tr>
<tr>
<td>$C_1$ &amp; $C_2$</td>
<td>Expressions used to calculate the buckling moment (Andrade et al 2007)</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Warping torsional constant</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between flange centroids (Trahair et al 2008)</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic modulus of steel</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Lower yield strength</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Upper yield strength</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus of steel</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Distance between flange centroids (Andrade et al 2007), equivalent to “$d$”</td>
</tr>
<tr>
<td>$I$</td>
<td>Interaction factor (Essa &amp; Kennedy 1994)</td>
</tr>
<tr>
<td>$I_y$</td>
<td>St Venant torsion constant of a cross-section (European practice)</td>
</tr>
<tr>
<td>$I_w$</td>
<td>Warping torsional constant (European practice)</td>
</tr>
<tr>
<td>$J$</td>
<td>Moment of inertia about y-axis, the minor axis (RSA and North American practice)</td>
</tr>
<tr>
<td>$J_z$</td>
<td>Moment of inertia about z-axis, the minor axis (European practice)</td>
</tr>
<tr>
<td>$K$</td>
<td>Torsional parameter of a segment (Trahair et al 2008)</td>
</tr>
<tr>
<td>$K$</td>
<td>Torsional parameter of a segment (Andrade et al 2007)</td>
</tr>
<tr>
<td>$k$</td>
<td>Effective length factor</td>
</tr>
<tr>
<td>$k_w$</td>
<td>Effective length factor for end warping restraint (Andrade et al 2007)</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Effective length factor for end rotations about the z-axis, minor axis (Andrade et al 2007)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of beam between lateral restraints, length of cantilever</td>
</tr>
<tr>
<td>$L_b$</td>
<td>Length of the backspan segment</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Length of cantilever (overhang) segment</td>
</tr>
<tr>
<td>$M_b$</td>
<td>Critical moment of the backspan segment that is free to warp (Essa &amp; Kennedy 1994)</td>
</tr>
<tr>
<td>$M_c$</td>
<td>Critical moment of cantilever segment that is free to warp (Essa &amp; Kennedy 1994)</td>
</tr>
<tr>
<td>$P_{cr}$</td>
<td>Elastic critical buckling load of a cantilever (Timoshenko &amp; Gere 1961)</td>
</tr>
<tr>
<td>$Q$</td>
<td>Critical buckling point load at free end of cantilever (Trahair et al 2008)</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Distance between the shear centre and the load applied (positive below the shear centre) (Trahair et al 2008)</td>
</tr>
<tr>
<td>$z_g$</td>
<td>Distance between shear centre and load applied (positive above the shear centre) (Andrade et al 2007)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Buckling parameter</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Dimensionless factor (Timoshenko &amp; Gere 1961)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Ratio of the smaller moment to the larger moment at opposite ends of the unbraced length</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Dimensionless load height parameter (Trahair et al 2008)</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>Equivalent moment factor ($C_b$ in American literature)</td>
</tr>
</tbody>
</table>
Equation 1 uses one effective length factor. It will be seen below that Andrade et al (2007) employ two different effective length factors for the torsional and warping stiffness terms.

Recent investigations to determine the $M_{cr}$ use different approaches to obtain the LTB capacity of cantilevers and overhang beams, and do not use the effective length factors published in the codes and by Ziemen (2010). Most notably were the investigations of Andrade et al (2007) and Trahair et al (2008) which are discussed below. Special purpose computer programs, such as PRFELB (Trahair et al 2008) are also available to compute $M_{cr}$.

### Andrade et al method

Andrade et al (2007) investigated cantilever beams that were either prevented from warping (NW) or free to warp (FW) at the support (in the latter case a cantilever with unrestrained flanges in the longitudinal direction). Thus the effect of an adjacent span was considered, but not the extent of the effect of the LTB stiffness of an adjacent span.

The method proposed by Andrade et al (2007) to determine the critical buckling moment is a rational approach, which extended the “3-factor method” to determine the critical buckling (FW) cantilevers, doubly symmetrical beams. The modified formula for free-to-warp (FW) cantilevers, symmetrical, and for bending about the major axis only, is defined as follows:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(k_w L)^2} \left[ \sqrt{\frac{k_z^2 I_w}{k_z} \frac{(k_w L)^2 GJ}{I_z \pi^2 EI_z} + (C_2 z_g)^2} - (C_2 z_g)^2 \right]$$

Where:

$C_1$ & $C_2$ are factors that depend on the warping restraint, the type of load, the distance between the shear centre and the load applied, and the torsional parameter (see Table 1)

$$z_g = \text{vertical distance between the shear centre (centroid) and the position of the load applied, measured as positive if the load is applied above the shear centre}$$

$$K = \frac{\pi}{L} \sqrt{\frac{EI}{4Gl}}$$

### Trahair et al method

Trahair et al (2008) presented an equation for overhanging beams that are free to warp at the support. This equation neglects the magnitude of warping restraint due to the length of the adjacent span. The method by Trahair et al (2008) approximates the buckling capacity of an overhang beam with a point load at the free end with the following equation:

$$P_{cr} = \frac{\sqrt{EI C}}{L^2}$$

Where:

$P_{cr}$ = elastic critical buckling load of a built-in cantilever

$\gamma_2$ = dimensionless factor depending on the ratio of $L^2 C / C_1$

$C = GJ$ and is the torsional rigidity of a beam

$C_1 = E C_w$ and is the warping rigidity of a beam.

### Timoshenko and Gere

The two methods provided above, though limited, did provide insight into the magnitude of buckling capacity expected for overhang beams. Both approaches have a non-dimensional term and a dimensional term $\sqrt{\frac{EI GJ}{L}}$, similar to the built-in cantilever (with point load at the tip shear centre) equation given by Timoshenko & Gere (1961):

$$P_{cr} = \gamma_2 \frac{\sqrt{EI C}}{L^2}$$

### Built-in cantilevers

Andrade et al (2007) and Trahair et al (2008) also provide equations for built-in cantilevers. The results derived from these equations could be compared to the equation given by Timoshenko and Gere (1961). A built-in W310 $\times$ 79 beam (equivalent to the South African designation 306 $\times$ 254 $\times$ 79 I) was analysed with different spans with a point load at the free tip centroid. To compare the $M_{cr}$ results from the three methods, effective length factors, $k$, were back-calculated from Equation 1 with the calculated $M_{cr}$ values. The effective length factor values, $k$, are plotted in Figure 1 against the dimensionless torsional...
For the case of an overhanging beam with a free tip:

$$I = -0.08 + 0.18 \frac{L_b}{L_c} - 0.009 \left( \frac{L_b}{L_c} \right)^2 \quad (9)$$

Two questions could be raised regarding these formulae provided by Essa and Kennedy (1994). Firstly, according to their simple FE models, the critical moment of the overhanging segment (top flange loading) was independent of the length of the segment, which seems unlikely. Secondly, for small ratios of backspan-to-overhang lengths $\frac{L_b}{L_c} < 0.5$ the interaction equation yields unrealistic values. These small ratios are perhaps outside the calibrated range of the authors; however, no limits regarding backspan length to overhang length ratio were specified in the paper.

The Essa and Kennedy (1994) equations are also applied in Table 2 (see page 9).

**PHYSICAL EXPERIMENTS**

Physical experiments were conducted to determine the buckling capacity of cantilever and overhang beams (Figure 2). Four built-in cantilevers were tested to ensure that the measurements recorded during testing were accurate. In total, 20 experiments were conducted on overhang beams. The beams used for the experiments were IPEA100 with a constant cantilever/overhang length of 2.5 m. The tests were repeated for both shear centre and top flange loading, with the backspan-to-overhang ratio ranging from 0.5 to 2.5, in increments of 0.5. Comprehensive details of the experimental program are given in Venter (2016).

**Overhang beam supports**

The overhang beam supports were designed to prevent lateral and torsional movement but to allow warping. To this end, rollers were used for the supports and adjustable vertical restraints were added next to the beam to prevent lateral
Figure 2 Built-in cantilever (left) and an overhang beam (right)

Figure 3 Design of external support for the overhang beam
and torsional movement. The vertical restraints were adjustable to account for the slight differences in flange widths for each beam.

The design of the external support (Figure 3) differed slightly from the internal support due to the upward reaction, and a second roller was provided above the beam. The second roller was adjustable to account for the difference in beam height for each beam. The external roller was also fixed to prevent longitudinal movement.

Loading of beams
The beams were loaded by a 1 000 litre water tank which was gradually filled and attached either at the shear centre of the beam or on the top flange. The load acted on the shear centre line, and top flange loading did not induce a measurable eccentric loading. The loading mechanism allowed for rotation and twisting of the applied load as the beam deflected and twisted when it buckled, also ensuring that the load applied remained essentially vertical during testing. Figures 4 and 5 show the design for the shear centre and top flange.
loading, respectively. A one-tonne calibrated load cell was attached to the bottom of the loading mechanism to measure the load. The data was logged and stored via a graphical logger (Graphtec Model GL220).

**Buckling loads from experiments**
The critical buckling moments, $M_{cr}$, obtained from the experiments are shown in Table 2. These loads were based on the maximum load obtained during the tests. Before testing, the beam dimensions were measured. It was noted that the beams were somewhat larger than the nominal dimensions given in the Southern African Steel Construction Handbook (SASCH 2013), but were within the allowable tolerances for hot-rolled sections as per the SASCH (2013).

The out-of-straightness of the beams, as well as the material properties, was measured. See the discussion below.

For reference purposes, $M_{cr}$ values were also calculated for certain cases. The following material values were used: $E = 200$ GPa and $G = 77$ GPa. The geometrical properties listed in the SASCH (2013) were employed.
For the built-in (fixed) cantilever with an unbraced tip and with shear centre point loading at the tip (Case Tb2a), the average test results are indicated. Theoretical values were calculated with the Timoshenko and Gere (1961) formula (Equation 4) given above, formulas given in the Andrade et al. (2007) and Trahair et al. (2008) publications, as well as the design code formula with an effective length factor = 0.8. The values are in reasonable agreement with a more conservative code value.

For the built-in cantilever with an unbraced tip and with top flange point loading at the tip (Case Tb2b), the average test results are indicated. Theoretical values were again calculated with the formulas given in the Andrade et al. (2007) and Trahair et al. (2008) publications, as well as the design code formula with an effective length factor = 1.4. The code value appears to be very conservative.

In order to provide a perspective on the \( M_{cr} \) values obtained from the tests for overhang beams with an unbraced tip and with shear centre point loading at the tip (Case Tb2c), the code was applied to separately investigate the capacity of the backspan and overhang segments. Backspan:

\[
\kappa = 0 \quad \omega^2 = 1.75
\]
\[
L_b = 3.625 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 5.92 \text{ kN.m}
\]

Backspan only

Overhang:

\[
L_b = 2.5 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 6.32 \text{ kN.m}
\]

Interaction

\[
L_b = 2.5 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 6.59 \text{ kN.m}
\]

Interaction

\[
L_b = 2.5 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 6.31 \text{ kN.m}
\]

Interaction

\[
L_b = 3.625 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 6.02 \text{ kN.m}
\]

Interaction

\[
L_b = 5 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 5.19 \text{ kN.m}
\]

Interaction

\[
L_b = 5 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 5.59 \text{ kN.m}
\]

Interaction

\[
L_b = 6.25 \quad L_c = 2.5 \quad \frac{L_b}{L_c} \quad M_{cr} = 4.82 \text{ kN.m}
\]

Interaction

As the interior support is considered to be hinged and the unbraced backspan (effective length factor = 1) is only loaded with the cantilever moment, \( \kappa = 0 \) and thus \( \omega^2 = 1.75 \). With an \( L_b = 3.625 \text{ m} \) the code gives an \( M_{cr} = 5.92 \text{ kN.m} \). Overhang segment: For a continuous cantilever with a “fork” support, the effective length factor = 1 (and \( \omega^2 = 1.0 \)), and with \( L_c = 2.5 \text{ m} \) the code gives \( M_{cr} = 5.09 \text{ kN.m} \).

For Case Tb2c, theoretical values were calculated with the formulas of Andrade et al. (2007) and Trahair et al. (2008) (see Equations 2 and 3) and are indicated in Table 2. The method of Essa and Kennedy.

### Table 2: Experimental and certain computed buckling moments

<table>
<thead>
<tr>
<th>(a) Fixed cantilever: shear centre loading</th>
<th>( L_c )</th>
<th>( M_{cr} )</th>
<th>(b) Fixed cantilever: top flange loading</th>
<th>( L_c )</th>
<th>( M_{cr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average test values</td>
<td>2.5</td>
<td>762</td>
<td>Average test values</td>
<td>2.5</td>
<td>6.85</td>
</tr>
<tr>
<td>Timoshenko &amp; Gere (1961)</td>
<td>2.5</td>
<td>789</td>
<td>Andrade et al (2007)</td>
<td>2.5</td>
<td>6.47</td>
</tr>
<tr>
<td>SANS 10162-1 (k = 0.8)</td>
<td>2.5</td>
<td>6.59</td>
<td>SANS 10162-1 (k = 1.4)</td>
<td>2.5</td>
<td>3.51</td>
</tr>
</tbody>
</table>

### Table 2: Continued

<table>
<thead>
<tr>
<th>(c) Overhang beam: shear centre loading</th>
<th>( L_b )</th>
<th>( L_c )</th>
<th>( L_b/L_c )</th>
<th>( M_{cr} )</th>
<th>Comment on buckling</th>
</tr>
</thead>
<tbody>
<tr>
<td>SANS 10162-1 (k = 1.0 and ( \omega^2 = 1.75 ))</td>
<td>3.625</td>
<td>2.5</td>
<td>1.45</td>
<td>5.92</td>
<td>Backspan only</td>
</tr>
<tr>
<td>SANS 10162-1 (k = 1.0 and ( \omega^2 = 1.0 ))</td>
<td>2.5</td>
<td>2.5</td>
<td>2.0</td>
<td>5.09</td>
<td>Cantilever only</td>
</tr>
<tr>
<td>Andrade et al (2007) – free to warp</td>
<td>2.5</td>
<td>2.5</td>
<td>2.0</td>
<td>6.30</td>
<td>Cantilever only</td>
</tr>
<tr>
<td>Trahair et al (2008) – free to warp</td>
<td>2.5</td>
<td>2.5</td>
<td>2.0</td>
<td>6.50</td>
<td>Cantilever only</td>
</tr>
<tr>
<td>Overhang, average test values</td>
<td>2.5</td>
<td>2.5</td>
<td>1.0</td>
<td>7.29</td>
<td>Interaction</td>
</tr>
<tr>
<td>Overhang, average test values</td>
<td>2.5</td>
<td>2.5</td>
<td>1.45</td>
<td>6.31</td>
<td>Interaction</td>
</tr>
<tr>
<td>Essa &amp; Kennedy (1994)</td>
<td>3.625</td>
<td>2.5</td>
<td>1.45</td>
<td>6.02</td>
<td>Interaction</td>
</tr>
<tr>
<td>Overhang, average test values</td>
<td>5.0</td>
<td>2.5</td>
<td>2.0</td>
<td>5.19</td>
<td>Interaction</td>
</tr>
<tr>
<td>Essa &amp; Kennedy (1994)</td>
<td>5.0</td>
<td>2.5</td>
<td>2.0</td>
<td>5.59</td>
<td>Interaction</td>
</tr>
<tr>
<td>Overhang, average test values</td>
<td>6.25</td>
<td>2.5</td>
<td>2.5</td>
<td>4.82</td>
<td>Interaction</td>
</tr>
</tbody>
</table>

### Table 2: Continued

<table>
<thead>
<tr>
<th>(d) Overhang beam: top flange loading</th>
<th>( L_b )</th>
<th>( L_c )</th>
<th>( L_b/L_c )</th>
<th>( M_{cr} )</th>
<th>Comment on buckling</th>
</tr>
</thead>
<tbody>
<tr>
<td>SANS 10162-1 (k = 1.0 and ( \omega^2 = 1.75 ))</td>
<td>3.625</td>
<td>2.5</td>
<td>1.92</td>
<td>5.92</td>
<td>Backspan only</td>
</tr>
<tr>
<td>SANS 10162-1 (k = 2.5 and ( \omega^2 = 1.0 ))</td>
<td>2.5</td>
<td>2.5</td>
<td>5.23</td>
<td>5.01</td>
<td>Cantilever only</td>
</tr>
<tr>
<td>Andrade et al (2007) – free to warp</td>
<td>2.5</td>
<td>2.5</td>
<td>5.33</td>
<td>5.01</td>
<td>Cantilever only</td>
</tr>
<tr>
<td>Trahair et al (2008) – free to warp</td>
<td>2.5</td>
<td>2.5</td>
<td>5.33</td>
<td>5.01</td>
<td>Cantilever only</td>
</tr>
<tr>
<td>Overhang average test values</td>
<td>5</td>
<td>2.5</td>
<td>2.0</td>
<td>5.01</td>
<td>Interaction</td>
</tr>
<tr>
<td>Overhang average test values</td>
<td>6.25</td>
<td>2.5</td>
<td>2.5</td>
<td>4.21</td>
<td>Interaction</td>
</tr>
</tbody>
</table>
(1994), which considers interaction buckling, was applied (see Equations 5 to 9) and the \( M_{cr} \) values indicated. Lastly, the average test results are shown and it could be observed how \( M_{cr} \) decreases with an increase in \( L_b \).

For overhang beams with an unbraced tip and with top flange point loading at the tip (Case Tb2d), the code was applied to investigate the capacity of the overhang segment. (The \( M_{cr} \) value for the backspan is not affected by the height of load application on the cantilever.) Overhang segment: For a continuous cantilever with a “fork” support and destabilizing loading, the effective length factor = 2.5 and with \( L_c = 2.5 \) m the code gives \( M_{cr} = 1.92 \) kN.m. Theoretical values were calculated with the formulas of Andrade et al. (2007) and Trahair et al. (2008) (see Equations 2 and 3) and are also indicated in Table 2. The code value again appears to be extremely conservative.

The scatter in the buckling loads is attributed to variations in initial out-of-straightness, beam sizes and material properties (see below) and, in addition, to the surface contact between the beam and the supports (rollers and vertical restraints) which caused additional friction when the beam was loaded. The additional friction in the flanges of the beam increased the warping resistance, causing the beam to resist a larger load before buckling occurred. Slight initial out-of-straightness also contributed to a lower critical moment. Photograph 1 shows the original and buckled shape of the back-span segment of an overhang beam.

Material properties

The material properties of the tested beams were determined (Venter 2016) via tensile (‘dog bone’) testing. Nine samples were cut from the web of an unloaded beam. The samples were loaded until fracture. The average lower yield strength \( f_y \) was 362.8 MPa, and the upper yield strength \( f_u \) was 377.8 MPa, according to the ISO 6892-1 (ISO 2009) standard. The calculated elastic modulus, via the tangent method, was 204.3 GPa. These values were used to calibrate the FE solid element models with the fixed cantilever experimental work.

Geometrical properties

Venter (2016) comprehensively documented the variations in beam dimensions and out-of-straightness. For such a small section, the allowable tolerances have a significant impact on the stiffness properties of the member. Most of the beams had flanges that were somewhat tapered. The slightest taper towards the web, combined with flanges and webs thicker than the nominal dimensions dramatically increases, for instance, the St Venant torsion constant. Maljaars et al. (2004, Figure 7) explains the significant role of the flange and web junctions on the St. Venant torsion constant.

FINITE ELEMENT METHOD

Finite Element (FE) analysis served two purposes in this study – firstly to expand the scope of the investigation, and secondly to obtain a relationship between the buckling capacities and the beam buckling parameters. The solid element models were first calibrated to the physical models to verify the accuracy and consistency of FE modelling when solving LTB problems, and then other size beams and overhang ratios were considered.

The FE program ABAQUS (2015) was used with the Buckling Analysis solver. The Buckling Analysis solver determines an Eigenvalue using the bifurcation method. An Eigenvalue is a load factor relative to the load applied to the model, which illustrates the ratio between the buckling load and the load applied. This FEM approach to obtain a buckling load factor would include the possibility of combined LTB and distortional buckling of the overhang beam (Bradford 1994).

Element properties

In the calibration exercise the measured properties of the steel were used to verify the FE models with the physical tests. The material properties used were: \( f_y = 362.8 \) MPa, \( E = 204 \) GPa and \( G = 77 \) GPa. For all further analysis on

<table>
<thead>
<tr>
<th>Table 3 Geometric properties of solid elements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property</strong></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Name</td>
</tr>
<tr>
<td>Type of element (nodes)</td>
</tr>
<tr>
<td>Aspect ratio</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Element size</td>
</tr>
<tr>
<td>Elements per cross-section</td>
</tr>
</tbody>
</table>
other beams other than IPE_{AA}100 the solid elements were assigned material properties of 350W steel (f_y = 350 MPa), with E = 200 GPa and G = 77 GPa. Residual stresses can reduce the critical moment of the beam below the theoretical elastic buckling moment. However, the effect of residual stresses decreases as the slenderness of the beam increases. Therefore, the FE models do not include residual stresses.

The geometric properties of the solid elements are provided in Table 3. Both hexahedral (20 nodes) and wedge (15 nodes) elements were used to improve the accuracy of the mesh, especially at the interfaces between the web and the flanges. Solid elements have six degrees of freedom per node. Comparative analyses with the thickness and width of the flange and web divided into multiple layers (mesh refinement) resulted in a negligible difference in the critical moment of the beam. Refinement of the mesh also did not improve the results. Therefore, to reduce computation time and the aspect ratio, only the web thickness was divided into two layers. Figure 6 shows the boundary conditions and the solid element IPE_{AA}100 model.

**Model dimensions analysed**

The elastic and inelastic ranges of buckling depend on the effective length of the beam. The ranges were defined by 0.67 \( M_p \) and 0.9 \( M_p \) for elastic and inelastic, respectively. The effective length factors provided by SANS 10162-1 (SANS 2011) were utilised to determine the length of the beam required for elastic LTB. The size and overhang lengths of the beams modelled using solid elements are provided in Table 4. The values in parenthesis refer to models with top flange loading, only, which have shorter lengths but remain in the elastic range (due to larger effective length factors).

**SENSITIVITY ANALYSIS**

The cross-sections of the tested IPE_{AA}100 beams were found to vary, and a sensitivity analysis was performed to assess the effect of varying the beam cross-section geometry on the buckling capacity. The sensitivity analyses were based on Equation 1 and were compared to the ideal fixed cantilever beam. For an IPE_{AA}100 cantilever beam with a length of 2.5 m, the buckling capacities were 6.59 kN.m and 3.51 kN.m for the shear centre and top flange loading, respectively. Table 5 presents the results of the sensitivity analyses.

The nominal dimensions and properties of the I-beam are based on a parallel flange section. It was established that some of the beams had thicker flanges than the nominal flange thickness, and also a taper. For sections with the thicker and tapered profiles of the flange this fact was deemed to have the largest influence on the buckling capacity. The dimensions of the beams tested were measured and were used to calibrate the solid element models, together with the measured material properties. The results of the calibrated models are provided and compared to the experiments in Table 6. With a maximum difference of 3.2%, it was found that the solid element models are an accurate numerical method to solve LTB problems. However, as previously stated, to achieve consistency the parametric study using FE solid element models was based on nominal dimensions and properties provided by the SASCH (2013).

**ANALYSIS OF RESULTS**

Analysis of solid element results

For a given backspan-to-overhang ratio, either the overhang segment or the backspan segment, or both segments together, dictate the critical buckling mode. For example, an IPE_{AA}100 beam with a 2.5 m overhang length underwent simultaneous

---

**Table 5 Sensitivity analyses for a 2.5 m long IPE_{AA}100 cantilever**

<table>
<thead>
<tr>
<th>Dimension / property</th>
<th>Change in dimension / property</th>
<th>Change in ( M_p ) (%)</th>
<th>Change in load (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web thickness</td>
<td>+0.2 mm</td>
<td>+2.1</td>
<td>+2.1</td>
</tr>
<tr>
<td>Flange thickness</td>
<td>+0.2 mm</td>
<td>+4.9</td>
<td>+4.9</td>
</tr>
<tr>
<td>Flange width</td>
<td>–3 mm</td>
<td>–10.6</td>
<td>–10.6</td>
</tr>
<tr>
<td>Beam height</td>
<td>+1.3 mm</td>
<td>+0.5</td>
<td>+0.5</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>+10 GPa</td>
<td>+5.0</td>
<td>+5.0</td>
</tr>
<tr>
<td>Flange profile</td>
<td>+0.1 mm</td>
<td>+23.5</td>
<td>+23.5</td>
</tr>
</tbody>
</table>

---

**Table 6 Comparing FE models to experimental results**

<table>
<thead>
<tr>
<th>Beam setup</th>
<th>Experimental result (kN.m)</th>
<th>FE model (kN.m)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever</td>
<td>6.79</td>
<td>7.01</td>
<td>+3.2</td>
</tr>
<tr>
<td>( l_f/L_c = 1.0 )</td>
<td>6.94</td>
<td>7.11</td>
<td>+2.4</td>
</tr>
<tr>
<td>( l_f/L_c = 2.0 )</td>
<td>4.94</td>
<td>5.06</td>
<td>+2.4</td>
</tr>
</tbody>
</table>

---

**Table 4 Size and lengths of beams analysed with solid elements**

<table>
<thead>
<tr>
<th>Beam designation</th>
<th>Length of overhang, ( L_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPE_{AA}100</td>
<td>2, 2.5, 3 and 3.5 m</td>
</tr>
<tr>
<td>IPE 200</td>
<td>3, 3.5 and 4 m</td>
</tr>
<tr>
<td>203 \times 133 \times 25</td>
<td>(2.2, 3.5, 3.5), 4, 4.5, 5, 5.5 and 6 m</td>
</tr>
<tr>
<td>305 \times 165 \times 40</td>
<td>(2, 3, 4), 4.5, 5, 5.5, 6, 7 and 8 m</td>
</tr>
<tr>
<td>406 \times 178 \times 54</td>
<td>4.5, 5.5 and 6.5 m</td>
</tr>
<tr>
<td>406 \times 178 \times 74</td>
<td>(2, 3, 4), 5, 5.5, 6, 6.5, 7 and 7.5 m</td>
</tr>
<tr>
<td>533 \times 210 \times 82</td>
<td>(5), 6 and 7 m</td>
</tr>
<tr>
<td>533 \times 210 \times 122</td>
<td>(3, 4, 5), 6, 6.5, 7 and 9 m</td>
</tr>
</tbody>
</table>
buckling in both segments if \( \frac{L_b}{L_c} = 1.0 \). This was noted in both the physical experiments and FE solid element modelling, as illustrated in Figure 7. The straight line in Figure 7 serves as a reference line to indicate buckling in both segments.

The results of the FE modelling of the 203 × 133 × 25 I-beam are shown in Graphs 1 and 2. The complete results of all the beams analysed are provided in Appendix A of Venter (2016). The discussions and conclusions that follow were based on all the analyses conducted, which apply to all beam sizes and lengths.

- Increasing the back span ratio \( \frac{L_b}{L_c} \), decreased the critical moment. However, for top flange loading, this observation was less profound.
- The buckling capacity became less sensitive to the overhang length \( L_c \) as the ratio \( \frac{L_b}{L_c} \) increased. This statement is only for shear centre loading.
- Top flange loading significantly decreased the buckling capacity of the beam.
- The reduction in buckling capacity, due to top flange loading, diminished as the overhang length \( L_c \) increased.
- These observations were consistent with all the beam sizes analysed.

Comparing FE results to experiments

To model the physical test results was a challenge, due to the scale of the experimental setup, the dimensional and material variations of the beams, the exact restraint conditions provided to the beam and, lastly, the method of load application (Venter 2016).

The experimental work (IPEAA100 with an overhang length of 2.5 m) and FE solid element ABAQUS models are compared in Graphs 3 and 4. Two FE models were done, firstly one with the nominal design dimensions and parallel flanges, and then a model with the measured cross-section dimensions (slightly larger) and tapered flanges. It was clear that the beam cross-section has a large impact on the buckling capacity of a beam, as justified by FE solid element modelling.

DESIGN EQUATION

To compare the FE models to the physical models, a single-sized beam with a fixed...
overhang length was used. However, to draw conclusions based on all the FE analyses conducted, a new approach was required to incorporate these additional parameters.

The closed-form solution of an I-section beam with lateral and torsional restraints, and under uniform bending, can be written as follows:

\[ M_{cr} = \frac{\pi L}{\sqrt{EI_y GJ}} + \frac{\pi^2 EC_w}{L^2} \quad (10) \]

The solution can be transformed to take the following term:

\[ M_{cr} = \gamma \frac{\pi L}{\sqrt{EI_y GJ}} \quad (11) \]

Where:

- \( M_{cr} \) = critical moment of the overhang beam
- \( \gamma = \sqrt{1 + K} \), a buckling parameter
- \( K = \frac{\pi^2 EC_w}{\sqrt{GJ L_c^2}} \) the torsional parameter.

The length of the overhang and the size of the beam were combined using the torsional beam parameter \( K \). With this approach, all of the models were comparable directly with a given loading condition.

The cantilever formula (Equation 4) given by Timoshenko and Gere (1961) was used as a basis on which the design equations were expanded. Equation 10 is the basic form of the design equation by Timoshenko and Gere (1961). The equation has two parts, a beam parameter relating the properties of the beam to the buckling capacity \( \frac{\pi \sqrt{EI_y GJ}}{L_c} \) and a non-dimensional buckling parameter, which takes into account the load height, support conditions and the backspan-to-overhang ratio.

\[ M_{cr} = S \frac{\pi \sqrt{EI_y GJ}}{L_c} \quad (12) \]

Where: \( S \) is a non-dimensional buckling parameter.

Rewriting Equation 10 provides a relationship between the critical moment \( M_{cr} \) and the nondimensional buckling parameter \( S \), given by Equation 13. However, by plotting \( S \) against \( K \) no discernable relationship existed between these two parameters. Instead, the non-dimensional part of the equation was ‘normalised’ by dividing it with \( K \) (Equation 14). Graphs 5 and 6
Graph 5 Normalised buckling parameter with shear centre loading

Graph 6 Normalised buckling parameter with top flange loading
illustrate the relationship between the ‘normalised’ non-dimensional buckling parameter \( S/K \) and the torsional beam parameter \( K \) for shear centre and top flange loading, respectively.

\[
S = \frac{M_{cr}L_c}{\pi \sqrt{EI_yGJ}} \quad (13)
\]

\[
\frac{S}{K} = \frac{M_{cr}L_c}{\pi K \sqrt{EI_yGJ}} \quad (14)
\]

The relationships between \( K \) and \( S/K \) were power functions, with different co-factors depending on \( L_b/L_c \) (Equation 15).

\[
\frac{S}{K} = AK^B \quad (15)
\]

Rewriting the equation to obtain the buckling parameter \( S \) and adding an adjustment factor \( C \), the design equation takes the form of Equation 16. The purpose of factor \( C \) was to ensure that the design equations were not too conservative (up to 13%) and overestimated by less than 1%.

\[
S = AK^{B+1} + C \quad (16)
\]

\( A \), \( B \) and \( C \) are second-degree polynomial functions of \( L_b/L_c \) and are defined in Tables 7 and 8. The non-dimensional buckling parameter \( S \) depends on the size of the beam, length of overhang, backspan-to-overhang ratio and the distance between the applied load and shear centre (load height).

To improve the accuracy of the design equations, the IPE AA100 beams were separated from the ‘universal’ beams. Universal beams refer to the beams typically manufactured in South Africa, i.e. the 203 × 133 × 25 i-beam. Plotting \( A \) and \( B \) against \( L_b/L_c \) revealed a quadratic function \( (Ax^2 + Bx + C, \text{ with } x = L_b/L_c) \) relating the parameters, as shown in Graphs 7 and 8 (for universal beams). The curves for
IPEAA100 were similar, with slightly different factors for A and B.

Comparing design equations

Table 9 illustrates the maximum and minimum differences between the design equations and the results obtained from the FE solid element modelling. These comparisons apply to all overhang lengths and backspan-to-overhang ratios analysed. A negative value implies a conservative result, while a positive value overestimates the buckling capacity. Note that the design equations were consistent regarding the size of the beam. In summary, the overestimate is always less than 1%.

For the specific case of $L_b/L_c = 2$ and $L_c = 6$ m the percentage under-determination of the equations' values of the FEM values against the minor radius of gyration, $r_y$, is shown in Graph 9 for the universal beam sections modelled. No trend could be detected.

Since Essa and Kennedy (1994) investigated the effect of the backspan on LTB capacity, it is worth comparing the FE results to their design method. Graphs 10, 11 and 12 illustrate the comparison for an IPEAA100 beam with a 2.5 m overhang and a 203 × 133 × 25 I-beam with a 4 m and 5 m overhang, respectively. As depicted in the graphs, the buckling capacity for top flange loading was highly variable when using the method proposed by Essa and Kennedy (1994). They stated that, for top flange loading, the critical moment of the overhang was independent of the overhang length. Thus, the buckling capacities were either over-conservative or overestimated, based on the size of the beam. For small backspan to overhang ratios ($L_b/L_c < 0.75$), their equation resulted in a decrease in buckling capacities, which is clearly incorrect as this is the opposite of what was observed in both types of FE analyses and physical experiments. The data of Essa and Kennedy (1994) in the graphs were according to the equations they published, but they clearly did not intend their formulae to be used for small $L_b/L_c$ ratios.

Design examples

Three examples are provided to illustrate how these design equations could be used. These examples illustrate the ease with which the critical moment of an overhang can be calculated. The examples are for an IPEAA100 beam (top flange loading) and a 406 × 178 × 74 I-beam (for both shear centre and top flange loading). The beams

<table>
<thead>
<tr>
<th>Beam designation</th>
<th>Maximum (%)</th>
<th>Minimum (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shear centre</td>
<td>Top flange</td>
</tr>
<tr>
<td>IPEAA100</td>
<td>-4.8</td>
<td>-71</td>
</tr>
<tr>
<td>IPE 200</td>
<td>-5.1</td>
<td>-70</td>
</tr>
<tr>
<td>203 × 133 × 25</td>
<td>-6.6</td>
<td>-12.8</td>
</tr>
<tr>
<td>305 × 165 × 40</td>
<td>-5.5</td>
<td>-12.9</td>
</tr>
<tr>
<td>406 × 178 × 54</td>
<td>-6.7</td>
<td>-11.8</td>
</tr>
<tr>
<td>406 × 178 × 74</td>
<td>-4.5</td>
<td>-11.1</td>
</tr>
<tr>
<td>533 × 210 × 82</td>
<td>-5.5</td>
<td>-9.7</td>
</tr>
<tr>
<td>533 × 210 × 122</td>
<td>-3.1</td>
<td>-9.3</td>
</tr>
</tbody>
</table>
were provided with lateral and torsional restraints at the supports, whereas the load was applied to the free end of the overhang beam. Table 10 illustrates the nominal properties of the two design beams according to the SASCH (2013).

**IPE\textsubscript{AA}100:**

From Table 8 for top flange loading:

\[ A = +0.069(0.5)^2 - 0.225(0.5) + 1.12 = 1.0248 \]
\[ B = +0.0121(0.5)^2 - 0.266(0.5) - 0.99 = -1.0928 \]

\[ K = \frac{\pi^2 ECw}{GLc^2} \]
\[ K = \frac{\pi^2 \times 200 \times 10^3 \times 0.272 \times 10^9}{77 \times 10^3 \times 7.33 \times 10^3 \times 2500^2} = 0.39 \]

\[ S = AK^{(B-1)} = 1.02475(0.39)(-1.09275+1) = 1.118 \]

\[ M_{cr} = S \frac{\pi \sqrt{EIyGJ}}{Lc} \]
\[ M_{cr} = 5300000 \text{N.mm} \]

\[ M_{cr} = 5.30 \text{kN.m} \quad \text{(ABAQUS = 5.5 kN.m)} \]

**406 × 178 × 74 I:**

From Table 7 for shear centre loading:

\[ A = -0.121(1.5)^2 - 0.2(1.5) + 1.89 = 1.3178 \]
\[ B = +0.044(1.5)^2 - 0.205(1.5) - 0.7 = -0.9085 \]
\[ C = +0.033(1.5) + 0.016 = 0.0655 \]

\[ K = \frac{\pi^2 ECw}{GLc^2} \]
\[ K = \frac{\pi^2 \times 200 \times 10^3 \times 610 \times 10^9}{77 \times 10^3 \times 642 \times 10^3 \times 6000^2} = 0.8226 \]

\[ S = AK^{(B+1)} + C \]
\[ = 1.3178(0.8226)^{(0.9085+1)} + 0.0655 \]
\[ = 1.360 \]

\[ M_{cr} = S \frac{\pi \sqrt{EIyGJ}}{Lc} \]
\[ M_{cr} = 278800000 \text{N.mm} \]

\[ M_{cr} = 278.8 \text{kN.m} \quad \text{(ABAQUS = 287.5 kN.m)} \]

The calculations were repeated for top flange loading.

Table 11 compares the proposed design equation with the ABAQUS FE solid element models, other publications and the current SANS 10162-1 (SANS 2011) method.
From Table 11 it could again be observed that the method of SANS 10162-1 (SANS 2011) produces extremely conservative values for top flange loading. The application of the Essa and Kennedy (1994) method did not produce realistic results for top flange loading.

CONCLUSIONS AND RECOMMENDATIONS

The experimental program posed several challenges, as discussed, which led to a significant scatter in the results. Nevertheless, the experimental work illustrated the buckling behaviour of overhang beams and produced useful data for benchmarking against other methods and the FEM results. The FEM method with solid elements (rather than shell elements) was successfully implemented for this buckling problem and also demonstrated the LTB behaviour of beams. The results were effectively benchmarked against the experimental work and other research data.

Many structural steel design codes, including SANS 10162-1 (2011), do not take the length of the backspan into account when calculating the LTB capacity of an overhang beam. Andrade et al (2007) and Trahair et al (2008) formulated equations for overhang beams by considering free-to-warp cantilevers, but neglected the effect of the length of the backspan. Certain methods have extended the solution of \( M_{cr} \) for single- and double-span beams for different boundary and loading conditions by applying a \( y \) factor, which is a function of more than the \( K \) parameter only. These extended solutions, however, do not include the backspan-to-overhang ratio \( L_b/L_c \). It was shown that the simplicity of the effective length approach leads to extremely conservative \( M_{cr} \) values for top flange loading.

This study investigated the effect of the backspan on overhang beams with supports which prevent lateral and torsional movement but allow warping. Based on the physical experiments and FE analyses, increasing the ratio of backspan to overhang reduces the lateral-torsional buckling capacity of an overhang beam. Also, increasing the overhang length has an adverse effect on the buckling capacity. For shear centre loading, the buckling capacity becomes less sensitive to the overhang length \( L_c \) as the ratio \( L_b/L_c \) increases. A proposed extended form of the factor \( y \) specifically formulated for overhanging beams is given in Equation 14. The proposed design method was validated using experimental investigations and verified by FE analysis.

While the limitations of the study are acknowledged in terms of the physical testing of only one beam size, support condition limited to lateral and torsional restraint, and only elastic buckling investigated, the authors believe that this work could lead to the start of more accurate assessment of the capacity of overhang beams. Further testing and analysis for other support, bracing and loading conditions could result in further refinement and in increasing the scope of the proposed design approach.

REFERENCES


Table 11 Summary of results for \( M_{cr} \) (kN.m)

<table>
<thead>
<tr>
<th>Beam</th>
<th>( \text{IPE}_{406} \text{A} 100 ) Top flange loading</th>
<th>( 406 \times 178 \times 74 ) I Shear centre loading</th>
<th>( 406 \times 178 \times 74 ) I Top flange loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed design method</td>
<td>5.3</td>
<td>278.8</td>
<td>179.8</td>
</tr>
<tr>
<td>ABAQUS FE model</td>
<td>5.5</td>
<td>287.5</td>
<td>199.0</td>
</tr>
<tr>
<td>Andrade et al (2007)</td>
<td>5.23</td>
<td>294.1</td>
<td>174.9</td>
</tr>
<tr>
<td>Trahair et al (2008)</td>
<td>5.33</td>
<td>295.3</td>
<td>184.9</td>
</tr>
<tr>
<td>Essa and Kennedy (1994)</td>
<td>–</td>
<td>272.7</td>
<td>–</td>
</tr>
<tr>
<td>SANS 10162-1 method</td>
<td>1.92</td>
<td>265.4</td>
<td>86.3</td>
</tr>
</tbody>
</table>
Economic benefit of ensuring uninterrupted water supply during prolonged electricity disruptions – City of Tshwane case study

J C Potgieter, C Herold, M van Dijk, J N Bhagwan

Mitigating the impact of electricity disruptions on water supply was investigated as a case study on the City of Tshwane. This study found that current institutional arrangements between electricity suppliers (Eskom), water service providers and water service authorities are insufficient. Therefore, a Risk Analysis and Mitigation Framework of Integrated Water and Electricity Systems, or RAMFIWES, was developed. Risks associated with water supply interruptions due to electricity disruption events were analysed. Risk categories that were addressed are short-term disruptions of less than one day, medium-term disruptions of up to a week and long-term electricity disruptions up to a month or even longer. The direct economic benefits of ensuring uninterrupted water supply in the event of electricity disruption events were analysed through cost versus benefit analyses. It was found that the direct benefit/cost ratio of supplying water during electricity disruption events is 5.6 for wet-industries and an exceptional 117 for other economic sectors in Tshwane. The overall benefit/cost ratio is 15.5. This benefit is possible at a low increase in the normal municipal bills of only 0.5%.

INTRODUCTION

The water supply sector is at the core of economic growth and social well-being. Water is indispensable to human survival – it is a critical component required for the generation of power, it is required for the growing of crops and it is a basic natural resource for daily existence. In the modern-day South African urban environment, without water there will be insufficient electricity, very little industry, no agriculture and no cities. Electricity disruptions can cause water supply interruptions which will have dire social and economic consequences for densely populated urban areas (ADB 2009).

Water and electricity are intrinsically linked – the one cannot be supplied without the other. This is especially true in South Africa where 85% of the country’s electricity is supplied by coal power stations (Pollet et al 2016), and urban water supply, especially in-land, necessitates high-lift pumping systems over great distances. This link is often referred to as the Energy-Water Nexus (Copeland & Carter 2017). Electricity is used in the water sector for pumping, treatment of raw water, distribution of potable water, collection and treatment of wastewater, and water discharge.

Until recently in South Africa, the electricity supply used in the water sector was considered safe, and the risk of electricity supply failure did not play a significant role in the design and operation of water supply and distribution systems. Load-shedding prompted the Water Research Commission (WRC) of South Africa in 2010 to conduct a high-level study of the effect of electricity disruptions, specifically load-shedding, on water supply (Winter 2011). The current study (this paper), also funded and initiated by the WRC, explores the implications in greater detail and takes account of new concerns that have arisen since 2011. These new concerns relate to the high risk of prolonged electricity disruption events that the water supply sector is currently not sufficiently prepared to mitigate. In recent years there have been numerous water supply interruptions in Gauteng due to localised electricity disruptions not linked to scheduled load-shedding, the most recent being the explosion at a City Power substation that resulted in an electricity disruption affecting Rand Water’s Eikenhof Pump Station (News24 2018). Subsequently, large parts of Gauteng, and even parts of North West, were affected by water supply interruptions. This electricity disruption was

CHRISTO POTGIETER (PrEng, MSAICE) works at Bigen in the company’s Water and Sanitation Sector. He graduated from the University of Pretoria with a degree in Civil Engineering in 2011, obtained his BEng (Hons) degree in 2016 and completed his MEng degree in 2018 in Water Resources Engineering.

Contact details:
Bigen Group,
PO Box 1994, Bloemfontein 9300, South Africa
T: +27 81 657 7504, E: christo.potgieter@bigengroup.com
Department of Civil Engineering, University of Pretoria
Private Bag X02, Hatfield 0028, Pretoria, South Africa

DR CHRIS HEROLD (PrEng, MSAICE) specialises in water resources and water quality model development and assessment. His models have found wide practical application in catchment and systems analysis studies that have saved South Africa billions of rands. He is the author of 44 technical papers published locally and overseas and is the principal author of over 100 unpublished technical reports. Chris has actively participated in over a dozen Water Research Commission studies, most of which he has led. He obtained his BSc, MSc and PhD degrees at the University of the Witwatersrand. He formed Umfula Wempilo Consulting in 2000.

Contact details:
Umfula Wempilo Consulting
PO Box 94578, Smanpark 2152, Johannesburg, South Africa
T: +27 21 459 5713, E: chris@ewd1.co.za

MARCUS VAN DIJK (PrEng, MSAICE) is a lecturer in the Department of Civil Engineering at the University of Pretoria (UP), and a Principal Researcher for the South African Water Research Commission. He obtained a degree in Civil Engineering from UP in 1996, an MEng in Water Resource Engineering in 2001, and is currently completing his PhD. He has compiled numerous technical reports and journal publications, and has presented at various conferences in the field of pipelines, hydropower generation, and water distribution systems. He is also a member of the Water Institute of Southern Africa.

Contact details:
Department of Civil Engineering, University of Pretoria
Private Bag X02, Hatfield 0028, Pretoria, South Africa
T: +27 12 420 3176, E: marco.vandijk@up.ac.za

JHONAT BHAGWAN obtained a Master’s in Tropical Public Health Engineering from Credos University, supported by degrees in Municipal and Civil Engineering. He is the Executive Manager of Water Use and Water Management at the South African Water Research Commission, overseeing around 150 research projects covering the management of drinking water and wastewater in the domestic, mining and industrial sectors, with particular focus on water supply and sanitation for low-income areas. He participated in the shaping of South Africa’s national water policy and legislation, and also directs the area of faecal sludge management (FSM), having presented an international series of FSM conferences.

Contact details:
Water Research Commission
Private Bag 103, Gezina, Pretoria 0031, South Africa
T: +27 12 761 9000, E: jay@wrc.org.za

Keywords: electricity disruptions, water supply, risk analysis, load shedding
The scope of the case study was confined to the City of Tshwane, which houses 3.1 million residents (CoT 2016). The city generates an annual Gross Domestic Product (GDP) of R202 billion (Tshwane Economic Development Agency 2015) and has an average water demand of 843 Mℓ/day (GLS Consulting 2017) which includes 193 Mℓ/day losses (CoT 2015a). It

As part of the Tshwane case study it was found that the current level of preparedness to mitigate the impact of electricity disruptions on water supply is more suited to accommodate short-term electricity disruption events (e.g. load-shedding with a duration of less than one day). There is a lack of preparation for medium- to long-term electricity disruption events. Therefore, medium- to long-term duration electricity disruption events pose a high risk for Tshwane, from both economic and public well-being points of view.

No guidelines, frameworks or government policies to aid electricity suppliers, WSPs and WSAs in the development of Disaster Risk Management Plans to mitigate the impact of electricity disruptions on water supply were found (the study concluded that there are none). Subsequently the Risk Analysis and Mitigation Framework of Integrated Water and Electricity Systems (RAMFIWES) was developed. The Tshwane case study was also used to test the part of RAMFIWES that deals with WSPs and WSAs (refer to Figure 1 for the outline of RAMFIWES).

The scope of the case study was confined to the City of Tshwane, which houses 3.1 million residents (CoT 2016). The city generates an annual Gross Domestic Product (GDP) of R202 billion (Tshwane Economic Development Agency 2015) and has an average water demand of 843 Mℓ/day (GLS Consulting 2017) which includes 193 Mℓ/day losses (CoT 2015a). It
goes without saying that addressing high real-water losses in the distribution system to decrease water demand is an essential part of ensuring uninterrupted water supply during electricity disruptions.

Some 81% to 86% of Tshwane’s water supply is derived from Rand Water and Magalies Water (CoT 2015b); the rest being supplied from own sources at Rietvlei Dam, Roodeplaat Dam and various dolomitic springs and wells (GLS Consulting 2017). Figure 2 illustrates the water network, including the WSPs’ water infrastructure.

**METHODOLOGY**

Background information on Tshwane’s demographics, economic activity, and water and electricity infrastructure was obtained through consultation with officials from Tshwane’s water and electricity departments and from various readily available data sources (such as Tshwane’s Integrated Development Plan and its annual financial reports) (CoT 2016; CoT 2015c).

Various electricity supply disruption events that would result in water supply interruptions in Tshwane were identified and grouped according to their areal extent, duration and probability of occurrence. These risks were based on literature sources and discussions with representatives from Eskom, Tshwane and Rand Water.

The impact of these events on water supply were assessed by considering three different Tshwane supply areas ranging in size from:
- a small residential area
- one of Tshwane’s six regions comprising mixed residential, commercial and industrial water uses
- the whole of Tshwane.

One-day, seven-day and thirty-day durations were examined for each selected size of supply area, giving a total of nine scenarios.

Mitigation options to sustain a minimum domestic water supply and protect most of the economic activity were identified and costed. Effective mitigation requires infrastructure development and operation, as well as effective implementation of institutional guidelines.

### Table 1 Summary of electricity disruption scenarios

<table>
<thead>
<tr>
<th>Scenario 1: Short-term (1 day)</th>
<th>Scenario 4: Medium-term (7 days)</th>
<th>Scenario 7: Long-term (1 month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description: Constantia Park Tower’s water supply interrupted</td>
<td>Description: Tshwane bulk water supply to region 6</td>
<td>Description: The entire City of Tshwane.</td>
</tr>
<tr>
<td>Population affected: 140 households (± 490 people)</td>
<td>Population affected: 173 000 households (± 600 000 people)</td>
<td>Population affected: 911 550 households (± 3.1 million people)</td>
</tr>
<tr>
<td>Area Annual Average Daily Demand (AADD): Residential – 355 kl/day</td>
<td>Area AADD: Residential – 120 Mℓ/day Industrial – 10 Mℓ/day Other – 30 Mℓ/day</td>
<td>Area AADD: Residential – 632 Mℓ/day Industrial – 55 Mℓ/day Other – 157 Mℓ/day</td>
</tr>
</tbody>
</table>
of mitigating the risks. A cost versus benefit analysis was done to determine the economic viability achieved. A cost versus benefit analysis was identified to mitigate the risks analysed in order to determine if the options identified to mitigate the risks would be practically feasible and economically viable.

The three electricity disruption areas and electricity disruption durations were analysed. A summary of the electricity disruption scenarios is given in Table 1 on page 21.

The approach used to analyse the impact of electricity disruption on water supply for the nine scenarios is described in Table 2.

### ELECTRICITY DISRUPTION SCENARIOS ANALYSED

Different types of electricity disruptions and their impact on water supply were analysed in order to determine if the options identified to mitigate the risks would be practically feasible and economically viable.

The three electricity disruption areas and electricity disruption durations were analysed. A summary of the electricity disruption scenarios is given in Table 1 on page 21.

The approach used to analyse the impact of electricity disruption on water supply for the nine scenarios is described in Table 2.

### MITIGATION OPTIONS

Loss of electricity supply has an immediate impact on the functioning of wastewater treatment works and raw sewage pumping systems, with consequent overflow of untreated effluent. There is also an impact on the ability to deliver water to elevated tanks supplying water to high-lying areas. Longer-term electricity supply disruption results in reduction or total loss of water supply even to lower-lying areas fed by gravity from reservoirs.

Mitigation options range from water supply by road tankers, provision of additional storage, provision of additional interconnectivity of pipe systems and the use of standby generators to make use of existing infrastructure (see Table 3 for more options). The latter (standby generators) proved to be the cheapest and most effective. This despite the fact that the already high water losses, increased by the elevated water pressure during periods of restriction, have to be fulfilled before water can be delivered to end users. Another important factor is that it is virtually impossible to provide a minimum water supply (25 ℓ/capita per day) to all domestic users without affording upstream users the opportunity to withdraw far more than their quota. Even in the highly unlikely event of 100% cooperation, it is impossible to implement effective water restrictions within the time spans of typical electricity disruption events.

The costs associated with mitigating the effect of each of the electricity disruption scenarios were scaled up (for scenarios that only considered smaller sections of Tshwane) and applied to the entire

<table>
<thead>
<tr>
<th>Water</th>
<th>Emergency electricity provision:</th>
<th>Communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>pool water</td>
<td>backup generators plus fuel</td>
<td>radio-relay systems</td>
</tr>
<tr>
<td>rainwater</td>
<td>shared backup generators</td>
<td>field wire</td>
</tr>
<tr>
<td>domestic water wells</td>
<td>rent backup generators</td>
<td>satellite communication systems with batteries, rechargeable batteries or solar panels</td>
</tr>
<tr>
<td>artificial static water supply sources</td>
<td>energy self-sufficient systems – photovoltaic panels, wind parks, sludge fermentation</td>
<td>short-wave radio gadgets with car batteries</td>
</tr>
<tr>
<td>open sources</td>
<td></td>
<td>Internal communication:</td>
</tr>
<tr>
<td>hand-turned pumps</td>
<td></td>
<td>alarm systems</td>
</tr>
<tr>
<td>high-level water storage tanks</td>
<td></td>
<td>radio broadcasting</td>
</tr>
<tr>
<td>mobile water tanks</td>
<td></td>
<td>flyers and brochures</td>
</tr>
<tr>
<td>water from neighbouring cities</td>
<td></td>
<td>personal communication</td>
</tr>
<tr>
<td>small water channels</td>
<td></td>
<td>loudspeaker announcements</td>
</tr>
<tr>
<td>bottled water</td>
<td></td>
<td>External communication places:</td>
</tr>
<tr>
<td>interconnectivity of pipe systems</td>
<td></td>
<td>city halls</td>
</tr>
<tr>
<td>water filtration</td>
<td></td>
<td>fire brigade houses</td>
</tr>
<tr>
<td>silver chloride pills</td>
<td></td>
<td>municipality houses</td>
</tr>
<tr>
<td>UV light irradiation</td>
<td></td>
<td>Risk communication channels:</td>
</tr>
<tr>
<td>boiling</td>
<td></td>
<td>news</td>
</tr>
<tr>
<td>distillation</td>
<td></td>
<td>specific events – change of the millennium</td>
</tr>
<tr>
<td>chlorination</td>
<td></td>
<td>seminars, workshops</td>
</tr>
<tr>
<td>plastic bags</td>
<td></td>
<td>environmental and political actions</td>
</tr>
<tr>
<td>manholes in public places connected to the sewage system</td>
<td></td>
<td>public incentives</td>
</tr>
<tr>
<td></td>
<td></td>
<td>platforms</td>
</tr>
<tr>
<td></td>
<td></td>
<td>books</td>
</tr>
</tbody>
</table>

Table 3 Implementation options to mitigate the effects of a water and power outage (adapted from Mank 2015)
Tshwane to obtain the total cost of mitigating each of the risks identified.

Finally, the scaled up costs were compared to the economic benefit of ensuring uninterrupted water supply to Tshwane.

COST / BENEFIT ANALYSIS
The cost of meeting minimum water supply requirements during electricity disruption events was compared to the economic and other benefits. The cost of mitigating each electricity disruption event type (load-shedding, distribution failure and blackout) was compared to the benefit, taking account of the probability of occurrence of the event.

The costs that will result from an electricity disruption event that causes water supply interruptions can be either direct or indirect.

Direct costs are due to interruption of the economic activity of the City of Tshwane. The benefit of mitigation is then the reduction in these costs. In such cases, it is possible to calculate the net benefit, benefit/cost ratio and the present value net benefit.

For scenarios that do not have a direct benefit with which to compare costs, the economic analyses have been confined to calculating the levelised annual cost and comparing this with the volume of water normally supplied to all Tshwane consumers to calculate a unit cost per kℓ. This facilitates direct comparison with normal water accounts to place decision-makers in a position to assess the implications.

The following were considered for the cost-benefit analysis:
- The city’s water demand
- The city’s water supply
- The city’s wastewater treatment
- The type, probability and duration of electricity disruption events
- The direct and indirect cost of the electricity disruption event.

Mitigation options were aimed at meeting the minimum requirements for the following water uses:
- Basic minimum supply for domestic water use
- Prevention of spillage of untreated sewage
- Sustaining GDP-generating activities.

The following assumptions were made:
- A 30-year life has been assumed for the standby generators. Although the life of mechanical/electrical plant is normally taken as 15 years (DPLG 2009), a longer life is feasible, since the plant will seldom be used, and then only for a relatively short duration.
- For similar reasons, the annual maintenance cost for generators has been reduced to 1% of capital.
- An annual net discount rate of 3% has been assumed. This value is considered appropriate for a large entity like Tshwane, since the net discount rate should reflect the net rate (i.e. after inflation) that Tshwane can expect to earn on a similar investment if it did not invest in the infrastructure required to ensure uninterrupted water supply during electricity disruption events.

The economic outlook period has been taken as the recurrence interval of each power outage event. Where necessary, the remaining value of capital works at the end of the outlook period has been credited before being discounted to a present value. A straight-line depreciation of capital over the life of the works has been used for this purpose.

In instances where there are no quantifiable direct economic benefits, such as meeting the minimum water supply requirement for domestic users, or prevention of sewage overflow, the outlook period has been set equal to the life of the capital works.

Costs have been expressed as cents per kℓ of normal billed consumption for comparative purposes. In the cases of scenarios covering power outages in small and medium-sized areas, the costs were scaled up to cover the whole of Pretoria.

Table 4 shows the estimated recurrence intervals for each scenario.

Interessingly, some of the severest blackout events are the easiest to assign RIs to. For example, the massive solar flare (the Carrington event) that occurred in 1859 (Cliver & Dietrich 2013) is well
documented, and two lesser events that are still considered to be of sufficient magnitude to hold grave consequences for modern electricity supply systems also occurred in the late 19th and early 20th centuries, when electricity generation and supply systems were still in their infancy. The possibility of the detonation of an Electro-Magnetic Pulse (EMP) device above the ionosphere is also not implausible, since both the United States and the USSR tested such devices in 1962, with devastating results. Considering that between 1945 and 1980 over 500 nuclear weapons tests were conducted (Beck & Burton 2002), the prospect of any nation which possesses nuclear weapons detonating such a device in the upper atmosphere is an all too real possibility.

**Basic minimum domestic water supply**

Meeting the basic minimum water requirement of residents is a non-negotiable cost. The cost, in terms of both human suffering and economic collapse, is simply too massive to ignore. A basic minimum domestic supply of 25 ℓ per capita per day has been used (CSIR 2005). However, achievement of this is assumed to require a supply at the top end of the distribution system of twice this amount to account for users higher up the system being able (even unconsciously) to abstract well above their quota, which would leave downstream residents with no water at all. The calculated minimum water to be supplied to domestic consumers during electricity disruptions is 451 Mℓ/day at 50 ℓ per capita per day (this includes real-water losses which will have to be supplied additionally).

Pumping water into elevated towers and reservoirs is considered to be part of meeting the basic minimum domestic water supply. The cost requirements for each scenario are shown in Table 5.

It is important to note that, once the most severe event has been catered for (Scenario 9), all of the capital and capital maintenance costs for the lesser events are automatically covered. However, the operating cost for each of the lesser events must be divided by its RI and accumulated to arrive at the total annual operating cost, which is added to the discounted capital and capital maintenance cost. Hence the additional charge to be borne by domestic water users paying for their services would come to 4.4 c/kℓ, which amounts to an increase of 0.44% (assuming a water tariff of R10/kℓ (Province of Gauteng 2017)). This is a small price to pay for protection against the social and political consequences of a national blackout.

There is a strong likelihood of violent social upheaval inherent in a national blackout. Moreover, such an event also has a high probability of occurrence (1:25 year RI for a seven-day outage and 1:155 RI for a 30-day outage, giving a combined RI of 1:22 years). That represents a 4.5% probability of occurrence in any one year and nearly a one in six chance of occurrence within the term of office of a politician. The small economic cost of protecting society against such a calamity pales into insignificance against such an enormous risk, which has an almost incalculable associated cost and high probability of occurrence.

**Maintaining industrial activity**

R22.6 billion of Tshwane's GDP is derived from the manufacturing sector, much of which is from wet industries (Tshwane Economic Development Agency 2015).

Since Rand Water supplies 76% of Tshwane’s water supply (CoT 2015b), the industrial water supply is only threatened by an event that shuts down power supply to both Tshwane and Rand Water. Since wet industries are highly dependent on water supply, process utilisation would be constrained by labour stay-aways and late arrivals due to transport difficulties during an electricity disruption event. Also, some users may not have enough power-generating capacity to maintain full operation. Accordingly, the assumption has been made that only 50% of industrial output could be sustained. This simplifying assumption has been made that only 50% of industrial output could be sustained.

### Table 5 Cost requirements for minimum domestic supply

| No | Description | RI (year) | Capital (mill R)# | Maintenance (mill R) | Operation (mill R) | 3% NDR*<br>Ann. Cost (mill R) | ∆ billing*<br>(c/kℓ) |
|----|-------------|----------|-------------------|---------------------|-------------------|------------------|----------------|-----------------|
| 1  | Small, 1 day| 1        | 0.000             | 0.000               | 0.000             | 0.000            | 0.000          | 0.000           |
| 2  | Medium, 1 day| 10       | 1.856             | 0.019               | 0.023             | 0.116            | 0.070          | 0.069           |
| 3  | Large, 1 day| 19       | 1.856             | 0.019               | 0.023             | 0.114            | 0.069          | 0.069           |
| 4  | Small, 7 day| 10       | 1.856             | 0.019               | 0.158             | 0.129            | 0.078          | 0.078           |
| 5  | Medium, 7 day| 30       | 1.856             | 0.019               | 0.158             | 0.119            | 0.072          | 0.072           |
| 6  | Large, 7 day| 25       | 108.329           | 1.083               | 9.205             | 6.978            | 4.236          | 4.236           |
| 7  | Small, 30 day| 50       | 1.856             | 0.019               | 0.676             | 0.127            | 0.077          | 0.077           |
| 8  | Medium, 30 day| 100     | 1.856             | 0.019               | 0.676             | 0.120            | 0.073          | 0.073           |
| 9  | Large, 30 day| 155      | 108.329           | 1.083               | 39.452            | 6.877            | 4.175          | 4.175           |
|    | Combined    |          | 108.329           | 1.083               | 50.370            | 7.290            | 4.425          |                 |

**Notes:**
- * Net discount rate.
- + Required increase in normal billing to customers based on average supply to paying domestic customers of 451 Mℓ/day (after deducting NRW water).
- # The capital included in the cost estimate is to mitigate the risks associated with the specific electricity disruption event (i.e. R1.856 million to ensure uninterrupted water supply for Scenario 2 includes backup electricity generation to high lying areas in the scenario area).
Table 6 Costs and benefits of maintaining 50% of industrial output

<table>
<thead>
<tr>
<th>Scenario</th>
<th>RI</th>
<th>Capital</th>
<th>Maintenance</th>
<th>Operation</th>
<th>Benefit/ event</th>
<th>3% net discount rate annual Benefit</th>
<th>Cost</th>
<th>B-C</th>
<th>B/C ratio</th>
<th>Δ billing +</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Description</td>
<td>Year</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
</tr>
<tr>
<td>6</td>
<td>Large, 7 day</td>
<td>30</td>
<td>28.303</td>
<td>0.283</td>
<td>2.405</td>
<td>216600</td>
<td>7.220</td>
<td>1.807</td>
<td>5.413</td>
<td>4.00</td>
</tr>
<tr>
<td>9</td>
<td>Large, 30 days</td>
<td>155</td>
<td>28.303</td>
<td>0.283</td>
<td>10.308</td>
<td>928100</td>
<td>5.988</td>
<td>2.298</td>
<td>3.689</td>
<td>2.61</td>
</tr>
<tr>
<td>Combined</td>
<td>28.303</td>
<td>0.283</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>13208</td>
<td>2.378</td>
<td>10.929</td>
<td>5.55</td>
<td>16.70</td>
</tr>
</tbody>
</table>

Note: + Required increase in normal billing to customers, based on average supply to paying domestic customers of 39 Mℓ/day (after deducting NRW water).

Table 7 Costs and benefits of maintaining 75% of other sectors’ output

<table>
<thead>
<tr>
<th>Scenario</th>
<th>RI</th>
<th>Capital</th>
<th>Maintenance</th>
<th>Operation</th>
<th>Benefit/ event</th>
<th>3% net discount rate annual Benefit</th>
<th>Cost</th>
<th>B-C</th>
<th>B/C ratio</th>
<th>Δ billing +</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Description</td>
<td>Year</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
<td>mill R</td>
</tr>
<tr>
<td>6</td>
<td>Large, 7 day</td>
<td>30</td>
<td>20.216</td>
<td>0.202</td>
<td>1.718</td>
<td>2.5786</td>
<td>85953</td>
<td>1.291</td>
<td>84663</td>
<td>66.6</td>
</tr>
<tr>
<td>9</td>
<td>Large, 30 day</td>
<td>155</td>
<td>20.216</td>
<td>0.202</td>
<td>7.363</td>
<td>11051.3</td>
<td>71299</td>
<td>1.281</td>
<td>70018</td>
<td>55.6</td>
</tr>
<tr>
<td>Combined</td>
<td>20.216</td>
<td>0.202</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>157252</td>
<td>1.339</td>
<td>155913</td>
<td>117.4</td>
<td>0.833</td>
</tr>
</tbody>
</table>

Note: + Required increase in normal billing to customers, based on average supply to paying domestic customers of 45 Mℓ/day (after deducting NRW water).
The large disparity between the benefits derived from these economic sectors, compared with those of the wet industries, is immediately apparent. Moreover, at only 30 Mℓ/day (25% of normal demand), the estimated minimum water requirement to sustain these activities is frugal. In terms of minimum water use these sectors contribute 19 times the economic contribution per unit of water used. Under normal circumstances this comparison is immaterial and the disparity in the overall economic contribution much smaller. But when water supply is severely constrained it is a much more important consideration. It is therefore considered extremely important to provide enough emergency water supply to sustain these sectors.

Underpinning this is the imperative to ensure a basic minimum water supply to domestic users. Without this the fabric of society will collapse, and with it all economic activity.

**Overall impact on potable water billing**

The overall average increase in potable water costs, taking account of the proportions of water supplied to each of the previously discussed three broad groupings (domestic, industrial and other economic sectors), comes to 5.0 c/kℓ, or 0.5% of normal paid billing. This is the estimated cost to secure a minimum basic water supply to domestic users, sustain 50% of industrial economic output and 75% of the output of other economically active sectors.

The associated overall annual cost is estimated at R11.0 million, yielding an annual benefit of R170.46 million. Hence the annual net benefit comes to R159.45 million, with an extremely high benefit/cost ratio of 15.5.

The very high benefit/cost ratio indicates rare resilience against variations in the estimates of cost and benefit. A sensitivity analysis was done to determine the robustness of the cost versus benefit analysis outcome; the following examples are given to illustrate this:

**Doubling the cost or reducing the benefit by half**

Doubling the cost estimate (to R22 million annually) or reducing the estimated benefit by half (to R85 million annually) would still yield a benefit/cost ratio of 7.7.

**Doubling the cost and reducing the benefit by half**

If the estimated cost of mitigation were doubled and the benefit were halved, the benefit/cost ratio of 3.9 would still remain very attractive.

**Benefit/cost ratio break-even point**

For the benefit/cost ratio to break even (i.e. benefit/cost = 1), the annual benefit will have to be decreased by a factor of 4 and the annual cost estimate will have to be increased by a factor of 3.87.

**Prevention of raw sewage spillage**

Standby power generation to prevent the spillage of up to 584 Mℓ/day (DWS 2014) of raw sewage is considered necessary to protect the natural environment and informal users, and to obviate biological overloading of downstream water treatment works. The capital, annual capital maintenance and operating costs for the standby power generation plant required for each scenario are shown in Table 8.

The increase in cost has been expressed in c/kℓ of effluent treated. This has not been expressed as a percentage of normal billing for sanitation services, since the requisite data was not at hand. However, if it is assumed that the sanitation billing is about half of that for water supply, then the percentage increase in sanitation billing to paying consumers would be roughly 0.8%.

If a decision is taken to make the R119.2 million capital investments in standby power generation plant to prevent raw sewage overflows, then all nine scenarios would be covered.

**INDIRECT COSTS OF ELECTRICITY DISRUPTION EVENTS**

Indirect costs are expected to be more than the direct costs. For instance, if an electricity disruption event that causes water supply interruptions triggers major city-wide (or nation-wide) civil unrest, it can result in the following indirect costs:

- Halting of all economic activity during and after the water supply interruption
- Violent uprising
- Loss of human life
- Spreading of disease
Loss of infrastructure
Economic vulnerability due to global uncertainty in South Africa’s economy and emigration from the country. Violent regime change might shut down Tshwane’s economy for a year or more, resulting in an economic cost equal to its entire GDP of R202 billion.

CONCLUSIONS
The main conclusions drawn from this study are as follows:

- There are currently no frameworks, regulations or government policies that guide electricity suppliers (i.e. Eskom), Water Service Authorities and Water Service Providers on how to mitigate the impact of electricity disruptions on water supply.
- There appear to be insufficient arrangements between key role players (Eskom, Water Service Providers and Water Service Authorities) regarding the roles and responsibilities of each role player in the event of an electricity disruption. Cooperation and coordination between these role-players during an electricity disruption event would be very limited as a result of this.
- In accordance with South Africa’s constitution, measures should be put in place to ensure that the country’s citizens are supplied with water to meet the minimum needs in the event of an electricity disruption. This is a joint responsibility between Water Service Authorities and the Water Service Providers that supply them with water. This action is necessary regardless of the economic benefit of ensuring continuous water supply during electricity disruption events.
- The City of Tshwane has very few mitigation measures in place to ensure continuous water supply during electricity disruption events (specifically medium- to long-term events). Given the effect of medium- to long-term electricity disruption events on the city’s water infrastructure, these events would have devastating effects on the city’s economy and, more importantly, its citizens.
- The Risk Analysis Mitigation Framework of Integrated Water and Electricity Systems, or RAMFIWES, was developed to address the need identified for such a framework. RAMFIWES proposes a structured approach which can be used to mitigate the impact of electricity disruption on water supply.

The following conclusions are drawn from this case study’s review of the City of Tshwane’s water infrastructure and the outcome of the scenario analyses:

- The benefits of ensuring uninterrupted minimum water supply greatly outweigh the costs of ensuring uninterrupted water supply purely from a direct economic cost-benefit analysis perspective. The benefit/cost ratio of mitigating the risks posed by electricity disruption events on water supply is approximately 5.6 for the city’s wet industries and 117 for the city’s other economic sectors.
- Provision of a basic domestic water supply, retaining 50% of industrial production and 75% of the output of other economic sectors, would increase the current normal billing to paying consumers by 0.5%.
- The intangible risks associated with prolonged water supply interruptions (socio-economic impacts) will probably be of greater concern than economic inactivity due to water supply interruptions.
- Standby generators to prevent large-scale spillage of raw sewage would add 0.8% to the normal sanitation bills received by consumers.
- Reducing the risk of damage to Eskom’s power generating facilities and distribution network during a blackout is highly desirable.
- For short-term electricity disruption events it is crucial to ensure, firstly, that reservoirs and elevated towers are large enough to be able to supply at least two days’ AADD, and secondly, that operating rules for reservoirs and towers are adhered to in order to ensure that water levels are maintained within the fluctuation volume of the reservoirs/towers.
- For medium- to long-term electricity disruption events it is concluded that the volume of water stored in the city’s reservoirs and elevated towers was large enough to mitigate risks posed by electricity disruptions is less important. This is due to the fact that the volume of water stored in the city’s reservoirs and elevated towers will almost certainly run out during medium- to long-term electricity disruption events if water supply cannot be sustained.
- The City of Tshwane will, in addressing medium- to long-term electricity disruption events, mitigate all risks associated with short-term electricity disruption events, which means that the capital cost of mitigating medium- to long-term risks will also address the short-term risks.
- Backup power generators (both mobile and permanent) will require ongoing servicing and maintenance – this will have to be incorporated into the operational and maintenance schedules of the city’s water department.
- Alternative energy sources (such as solar panels or batteries) should be considered as part of further investigations if it is decided to provide backup power to mitigate the risk of electricity disruptions on water supply – this will have to be investigated in separate cost-comparisons between various backup power supply options during the preliminary design stage for providing backup power generators.
- Providing emergency storage capacity for sewage inflow in wastewater treatment works is more expensive than providing backup power generation at wastewater treatment works, hence emergency storage will not be practical for medium- to long-term duration electricity disruption events.
- The supply and delivery of fuel to the city’s water and sewer pump stations, and its water and wastewater treatment works will have to be planned (and secured via a contract or formal arrangement) to ensure fuel gets delivered in the event of an electricity disruption event. On-site fuel storage will need to be considered, since lengthy electricity outages will probably also disrupt fuel supplies and dislocate transportation.
- Water restrictions implementation and end-user buy-in will be critical to ensure that water supply to the city is not interrupted in the event of an electricity disruption event. The most effective way to restrict water use to domestic and commercial users during electricity disruption events will probably be to close reservoir and elevated tank outlet pipes, and only open the pipes at certain times of day (after getting community buy-in).
- Public buy-in and acceptance of all other water supply mitigation options for by the City of Tshwane will be crucial to avoid intangible risks associated with water supply interruptions (such as wide-spread civil unrest, loss of human life, economic meltdown and civil war). The public has to believe that the mitigations opted for are, firstly, put in place by the municipality in the
public's best interest, secondly, not occurring unnecessarily as the result of negligence by any of the parties involved (e.g. Eskom, Tshwane or Rand Water), and thirdly, that the implementation of the mitigating options is the best way to ensure continued economic activity in Tshwane.

ACKNOWLEDGEMENTS
This study was funded by the Water Research Commission whose support is acknowledged with gratitude. The study team also wishes to express their gratitude to the City of Tshwane's Electricity and Water & Sanitation Departments for making the necessary information available for the Tshwane case study.

REFERENCES


Comparing the factors of safety from finite element and limit equilibrium analyses in lateral support design

J-T Potgieter, S W Jacobsz

Soil-nails and anchors as means of lateral support in surface excavations require stability analyses as part of design. Generally, the acceptance criterion is some arbitrary ‘factor of safety’ which can be routinely computed using well-established principles of limit equilibrium analyses. These methods have been tested over many years and have been shown to be adequate for design. There is an increasing trend towards the use of the finite element strength reduction methods to determine factors of safety in lateral support design. Differences are often reported between factors of safety calculated using these methods. There is also a danger that engineers might use finite element modelling without full appreciation of the impact of choices and assumptions made in using the software.

This paper compares the results of limit equilibrium and finite element calculations to assess factors of safety. Under certain conditions the factor of safety from the finite element strength reduction technique is comparable with limit equilibrium methods, provided that the same failure mechanism is evaluated. In addition, in the case of anchors, the same capacity must be specified in both analyses. In defining in-situ stress states, the friction angle and Poisson’s ratio should be specified so as to not violate the yield criterion. Modelling parameters (mesh grading and boundary distances) were found to have relatively minor influences on the factor of safety for the strength reduction technique.

INTRODUCTION

Urban construction projects often involve deep vertical excavations requiring lateral support. The use of soil-nails and anchors, in combination with a shotcrete facing and/or soldier piles, are popular means of lateral support. During design, the stability of the supported excavation face requires assessment.

The design philosophy with lateral support is to prevent excessive deformation and failure, and to provide a sufficient margin of safety to adequately satisfy soil, material and loading variability (Lai Sang & Scheele 1999; BS 1995). Despite serviceability considerations often being as important as failure conditions (Simpson & Driscoll 1998), design procedures generally focus on stability calculations (i.e. failure conditions), and only limited guidelines are given for serviceability requirements (SAICE 1989; Clouterre 1991).

In order to carry out stability computations, in the past, limit equilibrium calculations were exclusively used to design lateral support systems. However, finite element modelling has become increasingly popular. Further advances have also been made with regard to three-dimensional (3D) modelling. However, 3D finite element modelling remains costly and not suitable for routine analysis of most problems. It is recognised that, even if the excavation has sufficient length to represent a plane strain problem, the failure mode would still be three-dimensional. However, in almost all cases, a two-dimensional plane strain analysis is conservative (Azzouz et al 1983; Mollahasani 2015; Duncan 1996).

The use of soil-nails has rapidly expanded since the 1980s (FHWA 2003). Despite considerable work since then, the design of soil-nail support is often still based upon simplified and conservative models. General consensus has been reached that soil-nails can be adequately modelled as tensile members under normal installation angles (Jewell & Pedley 1992; Pedley et al 1990; Bridle & Davis 1997). The pull-out resistance in many soils can be calculated as a function of the effective stress at the depth of the nails. However, Heymann et al (1992) have shown that in South African residual soils, the pull-out resistance is generally independent of depth.
Anchored lateral support systems have been implemented since the 1930s (FHWA 1999). The design of anchored lateral support systems is based on an acceptable working load according to the appropriate code of practice. Working loads are obtained by factoring the ultimate capacity of the anchor. Anchor fixed-lengths are often pressure-grouted to obtain sufficient bond resistance. Soldier piles are often incorporated in anchored systems and generally extend below the base of the excavation, improving stability by creating passive resistance which can be analysed with simple methods as proposed by Brooms (1968) or Wang and Reese (1986).

Various researchers have compared limit equilibrium methods and finite element modelling (Duncan 1996; Griffiths & Lane 1999; Cheng et al 2007; Tschuchnigg et al 2015). However, most of these studies were based on evaluating the factor of safety (FoS) for slope stability problems, with few examining soil-nailed and anchored lateral support systems.

The differences in calculated factors of safety from limit equilibrium and finite element strength reduction analysis are often not well understood by practising engineers. Inappropriate use of finite element techniques can contribute to catastrophic failures, e.g. the 2004 Nicoll Highway Collapse (COI 2005). Potts (2003) also cautions users regarding the complexities of finite element computations.

This study explores differences in factors of safety from simple, well-established limit equilibrium methods to finite element strength reduction methods for soil-nailed and anchored systems. Despite many advanced numerical soil models becoming available, the Mohr-Coulomb model still remains the most widely used model in geotechnical engineering practice and is the focus of this study (ICE 2012). If the same soil model is used and the same reinforcement is used, why should the FoS, i.e. the parameter governing the stability interpretation, be different?

**Definition of Factor of Safety (FoS)**

The stability of an excavated face can be expressed in terms of the ratio between activating and resisting forces (or moments). The design should provide for a certain margin of safety against instability. For this purpose, a factor of safety has to be defined. Various definitions of the factor of safety exist.

Figure 1 shows the forces acting on a single wedge failure mechanism behind an excavation face supported by a soil-nail/anchored system. Both the activating force (self-weight of the wedge, \( W \)) and the resisting force (the tension from the reinforcement, \( T \), and friction along the failure plane) have components parallel and perpendicular to the rupture plane. Equilibrium of forces parallel to the rupture plane is considered in stability calculations. Orthogonal components of the self-weight and reinforcement tension cause friction resistance along the rupture plane, opposing sliding. Figure 2 shows the closed force polygon with four principal force components parallel to the slope requiring consideration:

1. \( T_{ho} \) The parallel component of the nail/anchor tension force
2. \( T_{soil} \) The normal component of the nail tension multiplied by \( \tan \phi \)
3. \( W_{ho} \) The parallel component of the weight of the wedge
4. \( W_{soil} \) The normal component of the weight of the wedge multiplied by \( \tan \phi \)

In addition to these components, a surcharge could be included on the surface of the wedge. A cohesive strength component could also exist, resisting sliding on the rupture plane. Both are omitted for the sake of simplicity.

The FoS can be defined as the ratio of the forces opposing sliding failure along the rupture plane against the forces driving failure. The definition of FoS in Equation 1 is used in literature such as Sheahan and Ho (2003), FHWA (2003), and Babu and Singh (2011).

\[
\text{FoS} = \frac{cH + W\cos\beta\tan\phi + \sum_{i=1}^{n} T_i\cos(\beta + \alpha) + T_i\sin(\beta + \alpha)\tan\phi}{W\sin\beta} = \frac{C_{hy} + W_{soil} + T_{ho} + T_{soil}}{W_{ho}} \tag{1}
\]

All symbols are defined in Figure 1.

An alternative definition of the FoS in Equation 2 is used in BS (1989) and SAICE (1989). Here the parallel component of the tension reinforcement is considered as a negative driving force.

\[
\text{FoS} = \frac{cH + W\cos\beta\tan\phi + \sum_{i=1}^{n} T_i\sin(\beta + \alpha)\tan\phi}{W\sin\beta - \sum_{i=1}^{n} [T_i\cos(\beta + \alpha)]} = \frac{C_{hy} + W_{soil} + T_{ho}}{W_{ho} - T_{soil}} \tag{2}
\]

This rearrangement results in the cohesive and frictional terms appearing in the numerator. The FoS can be seen as the number by which \( c' \) and \( \tan \phi' \) have to be
In this formulation, the FoS is synonymous with the Strength Reduction Factor (SRF) used in calculating the FoS using finite element analyses.

Juran and Elias (1987) advocated placing the FoS on the reinforcement capacity. This definition, shown in Equation 3, is found in codes from the early work of Stocker et al. (1979), and Gässler and Gudehus (1981) (e.g. SAICE 1989).

\[ \text{FoS} = \frac{\text{T}_{\text{required}}}{\text{T}_{\text{provided}}} \]  

(3)

\( \text{T}_{\text{required}} \) is the capacity required on a potential slip surface to maintain equilibrium. Equations 1 or 2 can be rearranged by substituting \( T_i \) with \( T_{\text{required}} \)

\[ \text{T}_{\text{required}} = \frac{\text{Wsin} \beta - \frac{\text{cH}}{\sin \beta} - \text{Wcos} \theta \tan \phi'}{\cos(\beta + \alpha) + \sin(\beta + \alpha) \tan \phi'} \]  

(4)

Substituting Equation 4 in Equation 3 gives:

\[ \text{FoS} = \frac{\text{T}_{\text{provided}}[\cos(\beta + \alpha) + \sin(\beta + \alpha) \tan \phi']}{\text{Wsin} \beta - \frac{\text{cH}}{\sin \beta} - \text{Wcos} \theta \tan \phi'} = \frac{\text{T}_{\text{ff}} + \text{T}_{\text{soil}}}{\text{W}_{\text{ff}} - \text{C}_{\text{ff}} - \text{W}_{\text{soil}}} \]  

(6)

All of the above definitions are valid and appear in different codes of practice. Different definitions exist due to various authors attributing the variability to be considered in a design to different components. At failure, i.e. FoS = 1.0, Equations 1, 2 and 6 are identical. When a partial factor method is used, attributing variability to each component, a unique overall FoS would be specified.

The formulation of FoS in Equation 2 was used in this paper to compare different methods of analysis.

**Analysis methods considered**

Limit equilibrium methods are routinely used to analyse slope stability problems (Tschuchnigg et al. 2015). Finite element methods also provide possibilities to assess the stability of supported excavation faces. Four methods of analysis are discussed below:

**Wedge Analysis**

The first, and also the simplest, method is the limit equilibrium Coulomb wedge analysis. The failure surface is assumed to be a straight line with the exit point fixed at the toe of the wall. The angle of slip surface is then varied to obtain the minimum FoS. The soil shear strength is based on the Mohr-Coulomb failure criterion. The FoS considers force equilibrium parallel to the slip surface as shown in Figure 1 according to the FoS formulation in Equation 2. The wedge analysis was implemented in a spreadsheet based on the Sheahan and Ho (2003) trial wedge method.

**Method of Slices**

The second limit equilibrium method investigated is the Method of Slices (MoS). The Morgenstern-Price (1965) method was used in this study, solving both force and moment equilibrium, and evaluating the inter-slice forces by a predefined function defining the relationship between normal and shear forces. Analyses were carried out using SLOPE/W (part of the GeoStudio suite).

**Enhanced Limit Equilibrium Method**

Krahn (2003) suggested using a combined finite element and limit equilibrium approach to calculate the FoS. A finite element analysis is used to obtain the stress distribution throughout the soil continuum. A limit equilibrium analysis is then used to evaluate the FoS based on the finite element calculated stresses by comparing the mobilised shear stress to the maximum available shear stress.
strength. This is known as the Enhanced Limit Equilibrium (ELE) Method (Kulhawy 1969; Naylor 1982). The rationale behind the ELE Method is to obtain a more realistic stress distribution along a slip surface than that obtained from the conventional MoS. An important aspect to note is that reinforcement elements are only considered in the finite element stress calculation phase. The GeoStudio suite was used to carry out analyses with the ELE Method.

Finite Element Strength Reduction Technique

Finally, the finite element (FE) strength reduction technique was evaluated. Since Griffiths and Lane (1999) popularised the idea of using a strength reduction factor in finite element displacement analyses, an increase in the use thereof to calculate the FoS has been observed in industry (Tschuchnigg et al 2015). Cheng et al (2007) found that factors of safety from limit equilibrium methods compare well with finite element strength reduction methods for slope stability problems.

In the FE strength reduction technique, soil shear strength parameters $c'$ and $\tan \phi'$ are reduced by a single Strength Reduction Factor (SRF) until failure occurs as shown in Equation 7. Failure is defined as either excessive deformation or an inability of the software to reach a converged solution. In this study, the software package PLAXIS 2D (2016) was used to calculate the SRF.

$$\text{SRF} = \frac{\tan(\phi')}{\tan(\phi'_{\text{reduced}})} = \frac{c'}{c'_{\text{reduced}}}$$

The finite element method has the distinct advantage of computing deformation; however, advanced models such as the Hardening Model (Addenbrooke et al 1997) have to be applied to consider realistic deformations (Obrzud 2010). Although the authors recognise that deformations are no less important to a good design, the purpose of this study is to consider the stability parameter commonly used: Factor of Safety.

Material parameters

The soil parameters used for this study are summarised in Table 1. The material was assumed to be uniform, homogenous and isotropic. The parameters were selected to be representative of those typically used to model excavations in residual granite in Johannesburg, South Africa.

An elastic-plastic soil model with the Mohr-Coulomb yield criterion was used, as this is still the most widely used model in general geotechnical practice (ICE 2012). The stiffness parameters ($E'$ and $v'$) were estimated for a dense material based on Bowles (1996).

### Table 1 Assumed parameters for residual granite

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle $\phi'$</td>
<td>°</td>
<td>36</td>
</tr>
<tr>
<td>Cohesion $c'$</td>
<td>kPa</td>
<td>3</td>
</tr>
<tr>
<td>Unit weight $y$</td>
<td>kN/m$^3$</td>
<td>19</td>
</tr>
<tr>
<td>Poisson’s ratio $v'$</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>Stiffness $E'$</td>
<td>MPa</td>
<td>90</td>
</tr>
<tr>
<td>Angle of dilation $\psi'$</td>
<td>°</td>
<td>18</td>
</tr>
<tr>
<td>Soil model</td>
<td></td>
<td>Elastic-Plastic (Mohr-Coulomb yield criterion)</td>
</tr>
</tbody>
</table>

![Figure 3 Cross-section of soil-nailed lateral support of 8.5 m excavation](image)

The required bar diameter and soil-nail length were determined from a limit equilibrium wedge analysis with a minimum FoS of 1.5. Five rows of 6 m long nails of 20 mm diameter were used to stabilise the excavation, providing a minimum FoS of 1.58. The critical failure plane extended at 41° above the horizontal from the excavation toe. The design and reinforcement parameters are summarised in Table 2.

### Analyses Conducted

**Soil-nail supported excavation**

An 8.5 m deep soil-nailed excavation was analysed as shown in Figure 3. The major factors to be determined during design are the soil-nail bar diameter and length. The bar diameter governs the yield capacity, while the length determines the pull-out resistance.

The pull-out resistance is a function of the allowable soil-grout interface shear stress, the drilling diameter and the length of soil-nail behind the most critical slip surface. The allowable soil-grout interface stress was taken as 110 kPa, a conservative estimate appropriate for residual granites in Johannesburg. Considering a drilling diameter of 102 mm, this provides a unit pull-out resistance of 35 kN/m. The soil-nail bar yield stress of 500 MPa was assumed from BS (2005). A 100 mm thick shotcrete face was assumed to adequately support the excavation face between soil-nails (FHWA 2003).

The pull-out resistance, with the bar yield strength as upper limit. In the Wedge and MoS analyses, the magnitude of the nail pull-out resistance, given a certain rupture plane, is determined from the length of the
soil-nail behind the failure surface multiplied by the unit pull-out resistance. The MoS assumes that this force can be applied to the base of the slice through which the soil-nail passes. In the case of the wedge analysis, the nail force is considered, as explained under the factor of safety definition above.

For the Enhanced Limit Equilibrium Method analysed using the Geostudio suite, soil-nails were specified as beam structural members. The ELE Method is suitable for calculating stress states under working stress conditions. It then uses the stress state to calculate a factor of safety using the MoS. The ultimate pull-out resistance and nail-yield strength were not considered. Generally, reinforcement is modelled as an elastic material.

Fan and Luo (2008), and Shiu and Chang (2006) used PLAXIS to model soil-nailed structures using plate and interface elements. However, the interface strength is governed by the surrounding soil strength, governed by the effective stress in a drained case. Heymann et al (1992) showed that, in the stiff residual soils in Johannesburg, the pull-out resistance is generally not a function of effective stress and that unit pull-out resistances can be taken as constant. Soil-nails were therefore specified as a combination of embedded piles and plates as shown in Figure 4. Plate elements can be specified as elastoplastic materials with a maximum tensile capacity defining yielding, after which the material behaves as perfectly plastic. Embedded pile elements can be specified to have a maximum pull-out resistance. The length of the embedded pile section was specified so that the total pull-out resistance was equal to the plate-yield strength. A slip surface passing in front of the embedded pile element (e.g. Slip A, Figure 4) will therefore trigger a yielding failure, while a failure passing through the embedded pile section (e.g. Slip C, Figure 4) will result in a pull-out failure.

The shotcrete facing was modelled using beam and plate elements in Geostudio and PLAXIS respectively. The shotcrete facing was assumed to deform elastically.

Anchored excavation

In general, anchors have higher tensile strength and pull-out resistance than soil-nails. Prestressed anchors are also more effective at controlling movements, due to being an active system. Figure 5 presents a cross-section of the anchored excavation showing the anchor layout and other relevant parameters.

When designing anchors, the major outputs are the required anchor capacities and layout. The layout is specified in terms of anchor spacing (vertical and horizontal), and free- and fixed-lengths. The load in the anchors after stressing is referred to as the anchor’s working-load. Anchors typically have additional capacity in excess of the working load that is used to provide for proof testing, lock-off losses, creep, and safety factors on the material strength and pull-out resistance.

Pressure grouting significantly increases the load-carrying capacity of the fixed-length (PTI 1996; FHWA 1999). The pull-out resistance of the anchor fixed-length is not easily predicted and, for this reason, codes allow for the fixed-length to be proof-tested at a load in excess of the working load. In order to compute an FoS for a failure surface passing through the fixed-length, the following assumptions were made:

- The pull-out resistance varies linearly along the 6 m grouted length.

### Table 2 Soil-nailed excavation – design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral support design</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height/depth</td>
<td>m</td>
<td>8.5</td>
</tr>
<tr>
<td>Reinforcement provided</td>
<td>m</td>
<td>5 rows of 6 m length, 20 mm diameter nails</td>
</tr>
<tr>
<td>Installation angle below horizontal (ICE 2012)</td>
<td>°</td>
<td>10</td>
</tr>
<tr>
<td>Soil-nail vertical spacing (rows)</td>
<td>m</td>
<td>1.5</td>
</tr>
<tr>
<td>Soil-nail horizontal spacing (out of plane)</td>
<td>m</td>
<td>1.5</td>
</tr>
<tr>
<td>Soil-nail bar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield stress (BS 2005)</td>
<td>MPa</td>
<td>500</td>
</tr>
<tr>
<td>Bar capacity</td>
<td>kN</td>
<td>138</td>
</tr>
<tr>
<td>Single nail, area</td>
<td>mm²</td>
<td>314</td>
</tr>
<tr>
<td>Single nail, area at 1.5 m spacing</td>
<td>mm²/m</td>
<td>209</td>
</tr>
<tr>
<td>Steel modulus E</td>
<td>GPa</td>
<td>200</td>
</tr>
<tr>
<td>Grouted bar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grout diameter (4 inch drill bit)</td>
<td>mm</td>
<td>102</td>
</tr>
<tr>
<td>Soil-grout interface resistance (Heymann et al 1992)</td>
<td>kPa</td>
<td>110</td>
</tr>
<tr>
<td>Pull-out capacity</td>
<td>kN/m</td>
<td>35</td>
</tr>
<tr>
<td>Shotcrete facing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness (FHWA 2003)</td>
<td>mm</td>
<td>100</td>
</tr>
<tr>
<td>Concrete modulus, E (BS 1985)</td>
<td>GPa</td>
<td>25</td>
</tr>
</tbody>
</table>

### Figure 4 Modelling of soil-nails in PLAXIS
The total pull-out resistance was set equal to the yield capacity of the anchor (proof-load testing will be approximately 90% of the yield capacity).

For a failure surface passing through the anchors, the pull-out capacity can be determined using the portion of the fixed-length behind the failure plane. Following ASTM (1987), seven-wire, 15.2 mm diameter steel strands were assumed. An acceptable working load of 150 kN per strand was used (SAICE 1989; Parry-Davies 2010) together with an assumed shotcrete facing of 150 mm thick.

The required tensile force was determined using a limit equilibrium wedge analysis with a target FoS of 1.5. Anchor working loads were provided as shown in Figure 5, resulting in a minimum FoS = 1.51. Adequate free-lengths were provided to ensure that the critical FoS would pass through the free-lengths and not around the back of the anchors. The critical failure surface was determined at an angle of 62° from the horizontal, extending from the toe. The design and reinforcement properties are summarised in Table 3.

Model boundaries and meshing
Finite element analyses were carried out to assess factors of safety using the ELE and FSRF methods. The extent of the model, mesh resolution and grading and the element type must be specified.

RESULTS AND DISCUSSION

Sensitivity to various parameters
A parametric study was carried out to evaluate the influence of variations in the input parameters on the factor of safety.

Table 3 Anchored excavation – design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral support design</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height/depth</td>
<td>m</td>
<td>17</td>
</tr>
<tr>
<td>Reinforcement provided</td>
<td></td>
<td>3 × 300 kN and 2 × 450 kN and 2 × 600 kN</td>
</tr>
<tr>
<td>Total anchor force</td>
<td>kN</td>
<td>3 000</td>
</tr>
<tr>
<td>Installation angle</td>
<td>°</td>
<td>10</td>
</tr>
<tr>
<td>Anchor vertical spacing (rows)</td>
<td>m</td>
<td>~2.2</td>
</tr>
<tr>
<td>Anchor horizontal spacing (out of plane)</td>
<td>m</td>
<td>2.5</td>
</tr>
<tr>
<td>Anchor tendons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single strand ultimate strength (ASTM 1987)</td>
<td>kN</td>
<td>261</td>
</tr>
<tr>
<td>Single strand yield strength (ASTM 1987)</td>
<td>kN</td>
<td>235</td>
</tr>
<tr>
<td>Single strand working load (SAICE 1989; Parry-Davies 2010)</td>
<td>kN</td>
<td>150</td>
</tr>
<tr>
<td>Single strand working load at 2.5 m spacing</td>
<td>kN/m</td>
<td>60</td>
</tr>
<tr>
<td>Single strand area (ASTM 1987)</td>
<td>mm²/m</td>
<td>140</td>
</tr>
<tr>
<td>Single strand at 2.5 m spacing</td>
<td>mm²/m</td>
<td>56</td>
</tr>
<tr>
<td>Reinforcement modulus E</td>
<td>GPa/m</td>
<td>200</td>
</tr>
<tr>
<td>Bonded length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>6</td>
</tr>
<tr>
<td>Total pull-out resistance (per strand)</td>
<td>kN</td>
<td>235</td>
</tr>
<tr>
<td>Unit pull-out resistance (per strand)</td>
<td>kN/m</td>
<td>39</td>
</tr>
<tr>
<td>Shotcrete facing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness (FHWA 1999)</td>
<td>mm</td>
<td>150</td>
</tr>
<tr>
<td>Concrete modulus E (BS 1985)</td>
<td>GPa</td>
<td>25</td>
</tr>
</tbody>
</table>

Figure 5 Cross-section of anchored lateral support of 17 m excavation
A linear elastic-perfectly plastic Mohr-Coulomb model was used for both ELE and SRF Methods.

Figure 6 shows the variation in FoS with the number of elements in the finite element model of the soil-nailed wall (in the case of anchors, the findings were similar). For the ELE Method, the FoS is sensitive to the number of elements. A large number of elements is required when selecting a uniform mesh size. Refining the mesh around structural elements (graded meshing) significantly decreases the total number of elements needed. The ELE Method FoS was found to also be sensitive to the lateral model extent.

Using PLAXIS, the FE (SRF) Method was found to be less sensitive to the model size and the number of elements. This is most likely attributable to the use of higher order 15-noded triangular elements compared to the first order elements used for the ELE analyses.

Material and reinforcement parameters
Soil properties and typical design parameters were varied to assess the influence on the FoS. Table 5 shows the sensitivity of the FoS to several input parameters for both the 8.5 m soil-nailed and 17 m anchored excavations. A near-linear correlation was generally found between the FoS and tabulated parameters.

The Wedge and MoS analyses show good correlation with the FE (SRF) Method across a range of values. However, trends from the ELE Method differed from the

### Table 4 Summary of assumptions used in different methods of analyses

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Soil model</th>
<th>Failure mechanism</th>
<th>Reinforcement elements</th>
<th>Calculation procedure</th>
<th>Package used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wedge analysis</td>
<td>Mohr-Coulomb</td>
<td>Straight line with the exit point fixed at the toe of the wall.</td>
<td>Anchors and soil-nails applied as force causing reaction on failure wedge.</td>
<td>Limit equilibrium. The angle of slip surface is varied. The FoS considers force equilibrium parallel to the slip surface as shown in Figure 1 according to the FoS formulation in Equation 2 (Sheahan &amp; Ho 2003).</td>
<td>Microsoft Excel</td>
</tr>
<tr>
<td>Method of Slices (MoS)</td>
<td>Mohr-Coulomb</td>
<td>Circular. Entry and exit points defined; radii varied to obtain critical failure.</td>
<td>Anchors and soil-nails applied as force causing reaction on failure wedge.</td>
<td>Limit equilibrium. Circular failure surface divided into thin slices evaluating the inter-slice forces by a predefined function defining the relationship between normal and shear forces solving both force and moment equilibrium (Morgenstern-Price 1965).</td>
<td>GeoStudio SLOPE/W (2007)</td>
</tr>
<tr>
<td>Enhanced Limit Equilibrium Method (ELE)</td>
<td>Mohr-Coulomb</td>
<td>Circular. Entry and exit points defined; radii varied to obtain critical failure.</td>
<td>Applied only in FE phase. Anchors applied as active force. Soil-nails force mobilised as deformation occurs.</td>
<td>Finite element and limit equilibrium. A finite element analysis is used to obtain the stress distribution throughout the soil continuum. A limit equilibrium analysis is then used to evaluate the factor of safety based on the finite element calculated stresses by comparing the mobilised shear stress to the maximum available shear strength (Kulhawy 1969; Naylor 1982; Krahn 2003). Reinforcement elements are only considered in the finite element stress calculation phase.</td>
<td>GeoStudio SLOPE/W and SIGMA/W (2007)</td>
</tr>
<tr>
<td>Finite Element Strength Reduction Technique/Factor (FE SRF)</td>
<td>Mohr-Coulomb</td>
<td>Any shape. Failure is defined as excessive movement / steady state.</td>
<td>Anchors applied as active force (working load). Further force generated as deformation occurs. Soil-nail force mobilised as deformation occurs.</td>
<td>Finite element. In the FE strength reduction technique, soil shear strength parameters $c'$ and $\tan^\phi$ are reduced by a single Strength Reduction Factor (SRF) until failure occurs, as shown in Equation 7 (Dawson et al 1999; Griffiths &amp; Lane 1999).</td>
<td>PLAXIS 2D (2016)</td>
</tr>
</tbody>
</table>
other methods. Unlike the other three methods, the ELE Method showed significant changes in FoS when changing Young’s modulus, Poisson’s ratio and the in-situ stress state. Furthermore, changing the soil-nail length and bar diameter did not significantly impact on the FoS. The reason for the differences is that the ELE Method evaluates the FoS at a working state and not at failure. Since soil-nails are passive systems, the capacity of the soil-nails is not considered when analysing the working state. The parameters mentioned above affect the stresses at working state, and hence influence the FoS. The other

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit increase</th>
<th>Method of Analysis</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction Angle, $\varphi'$</td>
<td>1°</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Cohesion, $c'$</td>
<td>1 kPa</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Unit weight, $\gamma$</td>
<td>1 kN/m$^3$</td>
<td>-0.09</td>
<td>-0.06</td>
</tr>
<tr>
<td>Poisson’s ratio, $v'$</td>
<td>0.1</td>
<td>-0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td>Stiffness, $E'$</td>
<td>x10 MPa</td>
<td>-1.79</td>
<td>-0.17</td>
</tr>
<tr>
<td>Angle of dilation, $\psi'$</td>
<td>1°</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>In-situ stress ratio, $K_0$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit increase</th>
<th>Method of Analysis</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surcharge</td>
<td>5 kPa</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
<tr>
<td>Nail length</td>
<td>1 m</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>Nail bar cross-sectional area</td>
<td>100 mm$^2$</td>
<td>0.20</td>
<td>0.22</td>
</tr>
<tr>
<td>Anchor length (fixed-length remains unchanged)</td>
<td>1 m</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Anchor working load</td>
<td>10%</td>
<td>0.17</td>
<td>0.16</td>
</tr>
</tbody>
</table>

**Table 5** Changes in factor of safety for changes in soil and design parameters
methods evaluate the FoS at failure state. The ELE Method is therefore not recommended for calculating the FoS when reinforcement elements are involved.

**In-situ stress violations**

When carrying out finite element computations, the initial in-situ stress state must be specified. Several options exist, depending on the problem under consideration and the software used. Commonly, a gravity turn-on procedure is used where the weight of the soil is ‘switched on’. Vertical stresses are calculated from self-weight. For an elastic plane-strain problem with a horizontal soil surface, the horizontal stresses are a function of Poisson’s ratio, with the coefficient of lateral earth pressure at-rest defined as:

\[
K_0 = \frac{\nu}{1 - \nu}
\]  

Another common way to specify the in-situ horizontal stresses is by using Jaky’s (1944) empirical solution:

\[
K_0 = 1 - \sin \phi
\]

SIGMA/W, by default, uses Equation 8 to define the in-situ stress state, while PLAXIS uses Equation 9. Most software also allows the user to manually specify the in-situ stress state.

Using the Mohr-Coulomb model, the yield criterion is a function of the soil shear strength parameters, \(c’\) and \(\phi’\). It is therefore possible to specify the in-situ stress state using Equation 8 so that \(K_0\) is less than the active pressure coefficient (\(K_a\)). This violates the yield criterion. Poisson’s ratio must therefore not be defined indiscriminately without considering this aspect.

**Failure mechanisms**

An advantage of the FE (SRF) Method is the ability to determine a failure mechanism without making any *a priori* assumptions about its shape or position. The traditional trial wedge method typically assumes a straight line with the exit point specified at the toe of the wall, while the MoS generally assumes a circular failure surface.

The FoS calculated using the trial wedge method and method of slices was found to agree well. For soil-nails, the FE (SRF) Method generally provided a somewhat lower FoS compared to the limit equilibrium analyses across a range of variables considered. Conversely, for anchored walls, the FoS provided by the FE (SRF) Method was slightly higher than for the limit equilibrium methods. However, the trends were found to be similar, i.e. an increase in friction angle results in the same increase for the limit equilibrium and the FE (SRF) Methods. The difference in failure mechanisms (i.e. the shape of the failure surface) is the primary cause for the differences in FoS between limit equilibrium and FE (SRF) Methods.

**Soil-nailed excavation**

Figure 7 shows the incremental shear strain distribution at failure for the soil-nailed excavation. Clear bands indicate the mechanism calculated using the FE (SRF) Method. A marked decrease in theFoS from 1.58 to 1.37 is observed for a failure extending beneath the toe of the excavation. Figure 9 illustrates Rankine active and passive pressures, with the net positive pressure highlighted. For a shallow depth below the toe, the positive active pressure exceeds the resisting passive pressure.
explaining the reduced factor of safety associated with mechanisms extending below the excavation toe. When using a multiple wedge mechanism, the FoS calculated for a soil-nailed excavation agrees well with that calculated from the finite element strength reduction technique.

**Anchored excavation**

As for soil-nails, the differences in FoS observed for anchored excavations can be partially attributed to difference in failure mechanisms. An additional difference between the limit equilibrium methods and the FE (SRF) Method is associated with the capacity of the anchors. In the FE (SRF) Method, despite the working load being specified, the soil shear strength is reduced to such an extent that the anchor-yield capacity is eventually mobilised. Since the yield capacity is in excess of the working load, the working load becomes irrelevant when the SRF is calculated to failure. The limit equilibrium FoS calculation uses the working load in the anchors, while the yield capacity is generally used in FE (SRF) calculations.

Figure 10 shows good agreement between the FoS when a multiple wedge failure resembling the FE (SRF) mechanism is compared. In this scenario, the anchor-yield capacity was specified in both methods of analysis. Figure 11 demonstrates the same point – the working load was used in both methods, and material below the toe was specified as rock, forcing the failure through the toe of the excavation. Close agreement is seen between the FoS from both methods.

**Strength Reduction Technique – parameter insensitivity**

The major factor driving instability in a laterally supported excavation is the weight of the retained soil. Stability of the anchored excavation, with an adequate FoS, is ensured by providing reinforcement. Since an unreinforced system cannot achieve static equilibrium – i.e. there is a deficit in force/moment resistance – an increase in unit weight should theoretically further increase this deficit and hence decrease the FoS.

Figure 12 shows the influence of unit weight on the FoS calculated using the four methods considered. The finite element methods seem to be insensitive to a change in unit weight. The reason becomes apparent when examining the associated failure mechanisms illustrated in Figure 13. For steeply inclined failure wedges, the driving force (component of the soil weight parallel to the failure plane) exceeded the resistive force (frictional resistance along the failure plane). For flat failure surfaces, the driving force could be matched by mobilising more frictional resistance.

As explained above, the FE (SRF) Method uses the yield capacity of the anchors instead of the working load used by limit equilibrium methods. This increased anchor strength results in the critical rupture surface extending behind the fixed-lengths instead of through the free-lengths of the anchors. The shape and position of the failure mechanism clearly affect the sensitivity of the FoS to certain soil and design parameters.

Further parameter insensitivity in terms of a strength reduction analysis is illustrated below.
Figure 14 represents hypothetical loads mobilised against the back of a retained excavation face as a function of face movement. The net driving force from the soil and the resisting force of the soil-nails are plotted on the vertical axis. The net driving force from the soil is expressed in terms of the mobilised horizontal earth pressure coefficient.

Suppose the soil-nails were ‘wished into place’ and that the initial in-situ horizontal soil stresses are defined by $K_0$. Initially (at point $a$), no force is mobilised in the nails as no movement of the excavation face had occurred. As the excavation face is allowed to move, tensile load will mobilise within the nails and, at the same time, the horizontal earth pressure coefficient within the soil will begin to reduce from $K_0$ towards $K'$. As the face is allowed to move further, an equilibrium point will be reached where the driving force from the soil and the restraining force from the nails will match (point $b$).

The point of equilibrium, and hence the amount of movement to reach this state, depends on the stiffnesses of the soil and nails, as well as the initial in-situ stress.

The evolution of the stress state on a hypothetical rupture plane associated with the above scenario is illustrated using the Mohr circle diagram in Figure 15(a). The Mohr circles representing the initial stress state (point $a$) and the stress state at equilibrium (point $b$) fall below the failure envelope which is represented in Figure 14 by $K'_0$. The soil is not yet in a state of failure. Now, at point $b$, strength reduction commences and the SRF is gradually increased, reducing the soil shear strength as shown in Figure 15(b). Initially, as the shear strength reduces, the stress state in the soil is unaffected until the reducing failure envelope touches the Mohr circle defining the stress state at equilibrium (point $b$). From this point onwards, the horizontal stress (defined by $K'_0$) increases with...
the increasing SRF. This is accompanied by increasing yielding on the rupture plane.

As the horizontal stress exerted by the soil increases, the force within the nails will also increase to maintain equilibrium. Equilibrium will be maintained up to the point where the soil shear strength has been reduced to such an extent that the nails reach their capacity (pull-out or nail yield, point c in Figure 14). Beyond this point, equilibrium can no longer be maintained and failure occurs.

Figure 14 shows that the magnitude of the SRF at failure depends only on the reduction in soil shear strength required to transition between the initial active state (SRF = 1.0 in Figure 14) and a state that will cause the nails to fail (SRF = 1.42 in Figure 14). Note that the magnitude of the SRF at failure does not depend on the location of the point of equilibrium (point b). Using the FE (SRF) Method, the FoS is also independent of the initial stress ratio, $K_0$, the stiffness parameters, $E'$ and $v'$, or modelling of the construction sequence (affecting the stress distribution behind the excavation face). However, these variables are likely to alter the stress state at working conditions (point b).

**CONCLUSIONS**

Practising engineers often come across significant differences in FoS calculated by different methods when designing lateral support for applications such as deep excavations. A comparison of limit equilibrium and finite element methods for stability analysis of soil-nailed and anchored excavations was presented with specific relation to the factor of safety (FoS) outcome.

A parametric study shows that the influence of changes in variables on the FoS is significantly influenced by the shape of the failure mechanism. Using the soil unit weight as an example: for steeply inclined failure wedges, an increase in unit weight decreases the FoS; however, for flat failure surfaces, the opposite is true.

At shallow depth immediately below the toe, the positive active pressure exceeds the resisting passive pressure, explaining the reduced factor of safety associated with mechanisms extending below the excavation toe.

It was found that FoS from the traditional single trial wedge method and the method of slices compare well over the range of analyses conducted. Factors of safety from the enhanced limit equilibrium method do not compare well with those from other methods, because it evaluates FoS at working state. The ultimate capacities of the reinforcement and the soil are therefore not necessarily considered.

The finite element strength reduction method produces an optimised failure mechanism and no *a priori* assumptions are required regarding the failure surface. The finite element strength reduction method FoS is not influenced by stiffness variables, in-situ stresses or staged construction modelling (which affect the stress state behind the retained excavation face). The FoS is only influenced by the yield criteria of the various materials and the initial ( unreduced) material properties. Factors of safety from the finite element strength reduction method and limit equilibrium methods are comparable under the following conditions:

1. Consistent failure mechanisms have to be applied (e.g. using a double-wedge failure mechanism).
2. The same ultimate reinforcement capacities, at failure, have to be considered.
3. Soil strengths are considered at ultimate state, not working state stresses.

**Figure 14** Hypothetical loading path of soil and nails from in-situ stress to failure

**Figure 15** Mohr circles of stress for hypothetical loading path: (a) for working state equilibrium, (b) for SRF technique and failure state
4. The same formulation of factor of safety has to be established.

Much experience has been gained in recent times on the assessment of the stability of soil-nailed and anchored excavations using the finite element strength reduction method. However, the increased complexity involved with finite element modelling necessitates cross checks with simpler limit equilibrium methods.

ACKNOWLEDGEMENTS
The authors would like to thank Mr Shaun Nell of Terra Strata for the funding provided to support this project, as well as Verديcon Consulting Engineers and Mr Ken Schwartz for their input on technical aspects.

REFERENCES
Small-scale dispersed Green Infrastructure – a fitting civil engineering solution to stormwater quality improvement?

I C Brink

Stormwater quality has been researched for decades, but design innovation has stagnated. The design engineer is commonly faced with a large array of design options, each with complex mechanisms requiring specialised knowledge, often in fields that do not form part of civil engineering training. Adequately combining and establishing the necessary knowledge from these fields requires a practically valid design focus. An investigation into whether small-scale dispersed Green Infrastructure (GI) can serve as such a focus for stormwater runoff quality improvement was performed. It was found that small-scale GI application provides a dispersed and passive treatment response to the spatially diffuse nature of stormwater runoff. This technology has comparable efficiencies to other traditional stormwater structures, with the added advantage of being incorporable into existing infrastructure. It is, however, not without its share of disadvantages and knowledge gaps. Future research into many aspects ranging from data collection to implementation is warranted.

INTRODUCTION

Stormwater quality is currently seen as the leading remaining cause of poor water quality in natural systems (USEPA 2016). Whereas technologies towards treatment and management of point sources of water pollution such as domestic and industrial sewage have been thoroughly researched and developed in the past century, few practical technologies have been developed for the treatment of stormwater runoff. Although international attention has been given to this topic since the 1960s, research and practical design guideline development in this area have stagnated. Innovation in this field is slowing down, with few designs based on quantitative theoretical bases. Instead, prescriptive methods such as the USA BMP design methods, whereby drainage structure geometry is prescribed, are commonly used (Fassman 2012; Brink 2016).

Literature on stormwater management, both for quantity and quality, was published decades ago internationally, as well as in South Africa. For example, Wanielista (1979) included a chapter on water quality responses to nonpoint sources with case studies more than 30 years ago. In South Africa, more than 15 years ago, Schoeman et al (2001) identified the need for managing the quality of urban stormwater runoff to prevent the spread of diseases, control the costs of water purification and reduce threats to the aquatic environment. In 2003, Quibell et al stated that in South Africa “…nonpoint source pollution has not been effectively addressed on an extensive scale.” Although such texts identified the problem of stormwater runoff quality years ago, practical engineering design guidelines have not emerged. In 2006 the American Society of Civil Engineers (ASCE 2006) stated that “…the control of water quality in urban runoff is in its technical infancy.” Today, another decade later, not much has changed.

South Africa suffers from the same major impediments regarding the development of practical design methodologies as many other countries do. Most prominent amongst these are difficulties with obtaining data on pollutant concentrations and flows, as well as a large selection of published design methods with little or no reference to their theoretical bases. For example, Schoeman et al (2001) performed a review of South African urban runoff studies and found that, although more than 50 studies had been done locally from the...
late 1980s, very few contributed meaningful water quality data coupled with flow data. Additionally, standard and comparable sets of water quality constituents were not measured across case studies. Costs associated with chemical analysis were mentioned as a main factor in the paucity of measured data.

The paucity of pollutant data by itself is not the only impediment to stormwater quality improvement solutions. Design data must also be obtained with a specific design methodology in mind. Adaptation of stormwater quantity control structures to serve as water quality improvement structures leaves the designer with a very large choice of structures, each with its own largely unexplained mechanisms of water quality improvement. Each mechanism of each structure requires specialised knowledge. Basins function with highly complex sedimentation processes in highly varying flow fields during storm events; wetlands have bio-removal and chemical speciation as an addition to sedimentation; swales include filtration and sorption, as well as sedimentation, again with highly varying parameters such as flows, bottom roughness, etc. These complexities in removal mechanisms, brought by high variation in structure physics, are difficult to work with. The result has been a large body of prescriptive methods, as well as some more involved methods that utilise standard engineering concepts but do not address removal mechanism complexities or data acquisition difficulties, including the time, manpower and costs involved with data gathering over a sprawling urban area over a significant period of time.

Stormwater design approaches that include water quality design considerations abound internationally. These include the United States Environmental Protection Agency (USEPA) Best Management Practice (BMP), the Australian and South African Water Sensitive Urban Design (WSUD) and the United Kingdom Sustainable Drainage Systems (SuDS) approaches. The USEPA website, for example, lists design manuals for a large number of states independently. Such manuals are often substantial and contain prescriptive qualitative recommendations. Specific and scientifically based guidelines are, however, lacking. For example, the New York State Stormwater Management Design Manual (Centre for Watershed Protection 2010) only states that rain-gardens have pollutant treatment benefits. It is also typical to find guidelines that include phrases like “moderate” or “high” pollutant removal for specific structures such as swales and ponds (e.g. ASCE 2006; Wisconsin Department of Natural Resources 2003) or even exact stated percentage removals such as 80% TSS removal for various structures (Wisconsin Department of Natural Resources 2003), or, for example, 49% Total Phosphorus removal for Wetlands (SEMCOG 2008). The design engineer, however, needs more detailed methods and information that include the quantities and types of pollutants removed, the mechanisms of removal as part of mathematical models, types of plants to implement, etc.

Published prescriptive methods are common and often include prescriptive design suggestions such as percentage plant coverage and specified side slope gradients (ASCE 2006; Wisconsin Department of Natural Resources 2003). For example, the ASCE (2006) Standard Guidelines for Design of Urban Stormwater Systems describe swales as planted shallow trenches with side slopes flatter than 3:1 and specify that retention ponds should have plants covering 30% of the pond area. In cases where such methodologies do not provide an explanation of the reasoning behind the choice of such exact values it can be questioned whether they can be applied as a general standard in different situations.

The generalised use of other, seemingly more involved design methods may also be questioned. For example, some methods use the Water Quality Volume parameter to design for water quality improvement (SEMCOG 2008). In these procedures, a Water Quality Volume is catered for in a design and the designer is thereby satisfied that a certain percentage pollutant removal will occur in the structure. However, the choice of Water Quality Volume itself is arbitrary, and a mechanism of calculation involving a range of parameter concentrations and flows may be more scientifically justifiable. Also common to more involved methods is a requirement for difficult-to-measure parameters. For example, the ASCE (2006) advises that swales be designed using equations for open channel flow with a “… roughness coefficient suitably adjusted for grass or vegetation.” Data towards such requirements is difficult, if not impossible, for the design engineer to find in a project where the time allowed for design for environmental considerations of the project is based on the amount of money the design company will earn. This is typically relatively low for small stormwater control structures.

In South Africa, stormwater quality improvement has recently been revisited. There is currently a movement towards the acceptance of the Water Sensitive Urban Design (WSUD) philosophy. Within this, Sustainable Drainage Systems (SuDS) form the stormwater management component (Armitage et al 2013c). This philosophy has many aspects, but has been specifically defined by the authors of the “Water Sensitive Urban Design (WSUD) for South Africa – Framework and Guidelines” (Armitage et al 2013b) as designing the urban landscape with water sensitivity in mind. This framework provides broad philosophical guidance, but its aim is not yet to provide design specifics. In line with this, the SuDS component focuses on stormwater management rather than on specific design guidelines (Armitage et al 2013c).

Although current design approaches do not seem to have strong theoretical bases, they are in keeping with successful historical general trends in water quality design development. The activated sludge system, for example, was successfully used long before informative mathematical models were developed towards its optimisation (Mogens et al 2008). It is, however, time to move away from theoretically unsubstantiated design recommendations to scientifically based practical design methodologies.

The next step towards a practical implementation of a stormwater treatment philosophy in South Africa is to ascertain where research and development energy should be spent. Quibell et al (2003) state that, although South African law makes provision for protection of natural water sources, implementation is constrained by a lack of human and financial resources. Therefore, they argue that only through use of limited resources to best effect can the goals of the legislation be met.

For the design engineer to have confidence in a well-understood stormwater structure design method for water quality control where pollutant removals can be predicted accurately, he/she requires not only a valid set of data, but also a practical design focus. The use of Green Infrastructure (GI) for treatment of stormwater is gaining recognition. For example, the United States Environmental Protection Agency (USEPA) promotes GI as an ecological way to manage stormwater through promoting on-site treatment as a
way to include natural plant-based systems in the urban environment (Yang et al 2015). In some USA states, GI application is even mandated by water regulators as part of new urban development projects (Anderson et al 2016), and is otherwise gaining much ground internationally (Geberemarium 2017). For example, a variety of GI systems have been implemented in New York City from as far back as the 1990s and the city released its Green Infrastructure Plan in 2010 already (UN Water 2018). The focus on GI as a practical design solution for stormwater quality improvement, one on which scarce development and research resources can be spent, is further investigated here. This paper therefore contains the results of an investigation into whether small-scale dispersed Green Infrastructure (GI) is a valid design and research focus area for stormwater quality improvement in terms of relative pollutant removal efficiencies, as well as practical considerations towards implementation.

WHY SMALL-SCALE DISPERSED GREEN INFRASTRUCTURE (GI)?

The definition of GI varies with different texts. A central precept across literature is the use of vegetated infrastructure to deliver desired services (Fletcher et al 2015). A particularly insightful definition is provided by the US Watershed Management Group (2012) who defines GI as “…constructed features that use living, natural systems to provide environmental services, such as capturing, cleaning and infiltrating stormwater; creating wildlife habitat; shading and cooling streets and buildings; and calming traffic.” This definition highlights secondary benefits, as well as the primary benefit of stormwater quality improvement.

Commonly used stormwater quality control structures that also have the characteristics of GI, as defined (natural features, interconnected, etc) are swales (planted infiltration ditches), filter strips (planted areas over which stormwater can flow), planted retention (wet detention) ponds and artificial wetlands (ASCE 2006). Small-scale systems typically do not require additional land and can be incorporated within existing infrastructure boundaries, e.g. at roadsides, within household plot boundaries, etc. Therefore, retention ponds and wetlands are not considered to be small-scale GI. Other small-scale GI structures include bio-retention structures (water is captured and allowed to filter into a planted area with underdrains) and green roofs (roof areas utilised for planting and landscaping) (USEPA 2009; 2012; 2014).

The response to stormwater quality improvement must fit the problem logistics. Stormwater runoff has been described as “diffuse” in many texts. This single word aptly highlights the main problem with stormwater quality treatment. Stormwater is spread out, not concentrated. Conventional water treatment infrastructure, such as used in potable water and sewage treatment, have a single inflow point. All water is collected in a network and conveyed to treatment plants, which can then be operated and maintained cost-effectively. In the case of stormwater, the situation is very different. Actively treating a myriad of stormwater outflows into streams, rivers and oceans is a logistic dilemma, because re-routing all stormwater pipelines to treatment plants will involve great cost and effort. An alternative to such active treatment is passive treatment, i.e. systems that do not require regular active input for correct functioning. GI involves passive natural systems that can be dispersed across urban areas, and may therefore be a fitting response to the dispersed nature of stormwater runoff.

In addition to active/passive treatment response, an important physical constraint to infrastructure implementation is land availability. Municipalities are responsible for implementation of stormwater quality control infrastructure in urban areas where public land either has commercial value or is highly limited at stormwater outflow points. The response must therefore be small-scale to enable implementation at a multitude of points where space is limited. Small-scale GI, such as vegetated filter strips, bio-retention structure facilities and swales, fit this requirement.

SMALL-SCALE GI MASS REMOVAL EFFICIENCIES

An investigation into solids mass removal fractions for a range of water quality control structures was done with data obtained from the International Stormwater BMP Database (2016). Masses and negative removals were specifically included to augment a lack of emphasis in published literature in this area. The case studies all originate from the United States of America. Although they are therefore geographically displaced from the South African setting, they originate from urban settings in another industrialised country, supporting the use of the investigative outcomes towards informing future research in South Africa.

Total Suspended Solids (TSS) and Total Dissolved Solids (TDS) comprise all stormwater pollutants. Further refinement into constituent pollutants was not deemed necessary at this stage, but should form part of future research. Structures were chosen according to availability of both water quality data and storm volume data. These were:

1. Detention basins, defined as basins with no significant permanent pool between storm events
2. Retention ponds, defined as basins with a permanent pool of water
3. Bio-filter grass strips, defined as vegetated areas that accept sheet flow
4. Bio-filter grass swales, defined as shallow grass-lined channels with little bottom width
5. Wetland basins or channels, defined as structures with dense wetland vegetation (more than 50% in the case of a basin)
6. Bio-retention structures, defined as shallow, soil-engineered, landscaped areas with or without underdrain systems.

(Definitions obtained from Wright Water Engineers Inc and Geosyntech Consultants Inc 2016)

Out these structural types bio-filter grass strips, grass swales and bio-retention structures were considered to be examples of small-scale GI.

Water quality, flow and monitoring station data was downloaded in raw form. TSS and TDS flow-weighted composite Event Mean Concentration (EMC) data was extracted and matched with corresponding flow volumes per storm event. EMC data was multiplied with structured in and out storm volumes to determine estimated TSS and TDS masses. Positive and negative (outflow masses greater than inflow masses per storm event) fraction removals were calculated. The fraction removal calculation was as follows:

\[ \text{Mass Fraction Removed} = \frac{EMC_{\text{in}} \cdot V_{\text{in}} - EMC_{\text{out}} \cdot V_{\text{out}}}{EMC_{\text{in}} \cdot V_{\text{in}}} \]  \hspace{1cm} (1)

Where EMC is the Event Mean Concentration (in or out of the structure) per storm event and V is the storm volume (in or out of the structure) per storm event.
Statistics for the positive results, where outflow masses were greater than inflow masses per storm event, as well as the percentage of negative removal events, are shown in Figures 1 and 2. Fassman (2012) performed a similar study with data from the same database pre-2012, but focused on concentrations rather than masses. The use of the mass fraction has the advantage of allowing direct inflow and outflow parameter comparisons without the influence of unaccounted volumes that may have entered or exited the structure, such as base flow, seepage or overland flow, which influence concentration measurements.

From Figure 1 it can be seen that the bio-filter grass strips and retention ponds had the highest median TSS mass removal efficiencies (> 0.9). These were closely followed by bio-retention structures and bio-filter grass swales (> 0.75). There were large discrepancies in sample sizes available for the analysis, and the high bio-filter grass-strip removal result must be accepted with caution with a sample size of only 23 (N). Positive mass fraction removal medians were below 0.75 for the detention basins (0.73) and wetlands (0.58).

The bio-retention structure and swale results were comparable with those found in literature. Fassman (2012) found, based on effluent concentrations, that bio-retention structures and grass swales tended to provide better effluent TSS concentrations. Bio-retention structures have been shown to be very efficient at stormwater pollutant removals. Percentage removal efficiencies have been over 80% for the metals lead, copper and zinc, as well as TSS (TRCS & CVC 2010; Xiao & McPherson 2011, Anderson et al. 2016). Nutrient removals of phosphorus and nitrogen, however, have typically been low or even negative, possibly due to fertilizer application (TRCA & CVC 2010; USEPA 2009). Yang et al. (2015) investigated bio-swales for metals, nutrients and TSS removal efficiency. They found that the bio-swale system outperformed the traditional detention basin system in their study with more than 80% difference in removals of all pollutants.

Positive fraction removal ranges were very high, with most structure types showing a possibility of achieving nearly no (0) removals as well as near-perfect (1) removals of all inflow masses. An exception was the bio-filter grass strips with a minimum positive fraction removal of 0.31. Data spread was highest for the structures with the worst removals, namely detention basins and wetlands (IQR > 0.5) and lowest for the best-performing structures, namely bio-filter grass strips and swales as well as retention ponds (IQR < 0.25). This indicates that the worst-performing structures had highly variable positive mass removal efficiencies. This may further indicate highly diverse design and operating conditions for these structures.

The percentage of storm events that resulted in negative fraction removals, i.e. where outflow masses were greater than inflow masses, showed that the worst-performing structures in terms of positive fraction removals also performed worst in terms of negative removals. Detention basins had negative mass removals during 26% of the storm events measured. Wetlands had negative mass removals during a notable 60% of storm events measured. The best-performing structures also had negative removals with 8% for bio-filter grass strips and swales and 15% for retention ponds. This indicates that design of these structures should include thought towards preventing re-suspension of previously captured solids during successive storm events.

Like for TSS, the best-performing structures in terms of TDS median removals were bio-filter grass strips (0.79) and retention ponds (0.93) (Figure 2). All other structures had relatively poor median removals (~0.4). All sample sizes (N) were relatively small when compared to TSS with the largest sample size for detention ponds (N = 70). Results should therefore be read with caution.

Maximum positive fraction removal ranges were once again very high, with most structure types showing a possibility of achieving nearly no (0) removals as well as near-perfect (1) removals of all inflow masses. Data spread was similar for all structures (0.38 > IQR < 0.6) (refer to Figure 2). This once again indicates highly variable positive mass removal efficiencies, and highly diverse design and operational settings for all structures.

The percentage of storm events that resulted in negative fraction removals showed different trends than were found for TSS. The best-performing structure type in terms of median removals, namely retention ponds, had the highest amount of negative removals (80%). Most other
GI KNOWLEDGE GAPS

Substantial work towards GI implementation has been done and can form a foundation for further improvements in design. In South Africa, the South African Guidelines for Sustainable Drainage Systems (Armitage et al 2013b), recently made available, provide guidance towards quantity design considerations such as runoff estimation and infiltration design. Referrals towards case studies and documented removal capacities of different technologies including green roofs, bio-retention structures, filter strips, swales and wetlands are given. Additionally, a number of GI projects have recently been implemented in the USA and may be used as examples of good design practices. The reader is referred to the EPA Green Infrastructure technical assistance program projects (USEPA 2009; 2012, 2014).

Although much work on various design guidelines has been done, further research into specific infrastructure design details is required. GI performance has been substantially researched and documented; however, this knowledge has not been adequately linked to theoretical explanations of removal mechanisms. Design guidelines often contain broad statements about pollutant removal abilities. Statements such as “80% removal of TSS” and “50% removal of total Phosphorus” (Wisconsin Department of Natural Resources 2003) are included with no reference to the source of the information and little theoretical support for the statements made. It is interesting to note that many design guides have the aim to reduce TSS in stormwater runoff by 80% and the stated efficiencies of the prescribed stormwater control structures are also exactly 80% TSS removal. This does not engender much confidence in the theoretical base for the stated efficiencies. Additionally, design of GI structures is not only lacking in removal efficiency knowledge, but also in insight into volume prediction. For example, a recent attempt at finding scientifically based and useable design insights for bio-retention structures have yielded limited results (Davis et al 2012).

In South Africa, Burke & Mayer (2009) identified, through consultation with various municipalities, the development of user-friendly stormwater selection, design and management tools as a top research need. This is a good end goal, but it is necessary to establish a scientific understanding of pollutant mechanisms first. GI design methodologies do exist (TRCA & CVC 2010; HR Wallingford Ltd 2017; USEPA n.d.), but are lacking in depth. For example, Cording et al (2017) state that, although roadside bio-retention structure systems are becoming increasingly popular, installation of such systems is outpacing research on the subject.

GI system design is complex and requires not only typical engineering hydrology knowledge, but also insights into the functioning of natural elements. Such knowledge is not usually included in the typical training of civil engineers. Conversely, mathematically based approaches to infrastructure design are not usually included in the typical training of environmental scientists. Nevertheless, it is exactly such an amalgamation of knowledge from both fields that is required for the creation of a design science for GI. The contribution of plants, soil filtration and micro-organisms to pollutant removals has not been greatly explored by engineers. Although plant uptake and filtration are often seen as major removal
mechanisms, there are also indications that bacteria can be responsible for significant amounts of pollutant uptake. For example, Endreny et al. (2012) found reduction in nitrate and zinc concentrations in a bio-retention structure bacterial column study, while conversely, phosphates increased in some cases. Application of GI as an engineering solution to stormwater quality improvement therefore requires extensive research into species selection, pollutant uptake mechanisms and implementation specifically towards stormwater quality improvement.

Some research into species selection is under way in South Africa. Milandri et al. (2012) performed a study to assess the nutrient removal capabilities of nine locally occurring plant species in Cape Town, eight of which were indigenous. Compared to a soil-only control, they found that most species were efficient at $\text{PO}_4^{3-}$ and $\text{NH}_3$ removals, achieving removal rates of over 90% in some cases. The soil-only controls were on average only 70% efficient for these parameters. Additionally, the soil-only controls removed on average 22% of $\text{NO}_3^-$ while removals up to 88% were found with certain plant species. Identification of the most appropriate species to use for pollutant removals in urban applications, modelling of removal mechanisms by plants, soil and micro-organisms within the system, and determination of practical installation aspects require further research. This has also been identified by other researchers working in this area (Yang et al. 2015).

In addition to the lack of understanding of pollutant removal mechanisms, data acquisition remains an impediment to design. Any design requires input, but pollutants in runoff are difficult to estimate without access to site-specific data, which is expensive and time-consuming to acquire. In South Africa, Quibell et al. (2003) developed a non-point source assessment guide to aid designers in the selection of appropriate modelling tools. The main focus of the work was, however, on sediment runoff in agricultural areas and all models discussed required in-field knowledge of parameters, which is difficult to obtain and can be highly variable or qualitative. More research into this area is warranted.

**GENERAL ADVANTAGES AND DISADVANTAGES COMPARED**

A key advantage of GI as a solution to stormwater quality is that it can be integrated as a source control measure. This enables a dispersed response to the dispersed nature of stormwater. The ASCE (2006) state that source control is difficult to establish, because home owners are unlikely to limit application of pollutants such as fertilisers to their gardens. It is, however, possible to persuade not only home owners, but also municipalities that manage public open spaces, to consider site control measures such as application of water purifiers (specific plant types) and infiltration systems to their planted and open areas.

An apparent major disadvantage of GI is that it, as the name implies, includes living entities, most notably plants. Plants may grow prolifically in the right conditions. Regular maintenance of such infrastructure will therefore be required and may be seen as a disadvantage by municipalities or private owners. For example, Armitage et al. (2013a) noted that the Century City wetlands (Cape Town) seemed to be able to mitigate stormwater quality, but required much maintenance, including annual dredging, fish removal, bi-weekly invasive plant and algae removal and regular bird faeces removal. A counter argument is that the labour-intensive maintenance requirement could be used in public job creation initiatives. An additional disadvantage is that plants require water, which is a scarce resource in South Africa. This requirement may be mitigated by planting indigenous species that are adapted to the local rainfall patterns.

A wide range of auxiliary advantages is often touted for Green Infrastructure. These include protection of watersheds, improvement of outdoor experiences, and improvement of air quality (USEPA 2012), carbon storage and even climate change impact reduction (HR Wallingford Ltd. 2017). Not much direct evidence is provided to back up such statements. This wide focus can, however, be a disadvantage for design. It may distract from specific application towards the purpose of stormwater quality improvement. The application of the concept of GI in civil engineering towards the improvement of stormwater quality should have only one goal, namely the improvement of stormwater quality. The design engineer must not be distracted by this broad range of GI benefits, which may be turned into objectives by the human mind. In other words, including design thinking towards “air purification” or “climate change impact reduction” should be excluded from the stormwater quality application. These possible benefits can instead be included as secondary advantages.

Additional considerations include:

- **Cost:** The capital costs associated with Green Infrastructure are generally considered to be lower than conventional technologies, but may require more intensive maintenance (HR Wallingford Ltd. 2017; USEPA 2012). Conversely, specific applications such as green roofs may cause increased costs due to increased load carrying structural requirements, waterproofing, etc (TRCA & CVC 2010).

- **Public Safety:** A possible concern with the planting of bushes and trees in public areas is that areas for crime are created. Conversely, public green areas can also be seen as having health benefits towards stress reduction. Kondo et al. (2015) performed a study to ascertain the effects of green stormwater infrastructure on public safety over two years at sites in Philadelphia, USA, by relating crime (assaults, robberies, narcotics possession, etc) to areas with green stormwater infrastructure. No relationship was found for most crimes, including serious crimes (gun possession, assault, etc), or health indicators. Interestingly, the authors did find a reduction in narcotics possession in these spaces. Therefore, this study indicates little to no difference to the wellbeing, or lack thereof, for people who live close to green infrastructure spaces.

- **Nutrient addition:** Plants require nutrients such as phosphorus and nitrogen. These will need to be applied to new plant installations, probably as fertiliser, to ensure healthy plant growth. The very installation of GI may therefore add nutrients to the stormwater overflow. For example, EPA (USEPA 2009) found that green roofs from five sampled sites had more than 300% higher phosphorus in runoff than standard asphalt roofs, although the average value was still relatively low at 0.41 mg/L. Furthermore, bacteria may release phosphates from the soil medium. Endreny (2012) found increases in phosphorus concentrations in a bio-filter bacterial column study. This was attributed to phosphorus release during microbial decomposition of organic media.

- **Stormwater runoff volume reductions:** GI has been shown to not only reduce runoff volumes through infiltration,
but also through evapotranspiration (Schoeman et al 2001; USEPA 2012).

- Improved biodiversity: A study by Kazemi et al (2011) in Melbourne, Australia, where nine bio-retention swales were studied in comparison to lawn and garden-type green spaces found a higher invertebrate species richness in the swales. This indicated that GI, by utilising a variety of vegetation, can be used to increase living spaces for indigenous plants and animals within urban environments.

- Nuisance: An increase in nuisance elements such as standing water and mosquitoes (TRCA & CVC 2010) can, however, be mitigated through careful design.

CONCLUSIONS

Even though stormwater quality has been researched for decades, design innovation has stagnated. The design engineer is commonly faced with a large array of structure options, each with either prescriptive qualitative or more involved design methods that have difficult-to-obtain parameters. Such design methods commonly have claims of percentage removals with seemingly little scientific base. This is a result of the application of simplified approaches to highly complex problems. Each mechanism of each structure requires specialised knowledge, often in fields that do not form part of civil engineering training. Knowledge of not only complex fluid dynamics, but also within the fields of botany (plant uptake, species selection), microbiology (bacterial uptake) and geology (soil interaction) is required. Adequately combining and establishing the necessary knowledge from these fields within a limited research and development resource context requires a practically valid design focus.

Small-scale dispersed Green Infrastructure (GI) was found to be a valid civil engineering approach to stormwater runoff quality improvement and can serve as a practical design focus point. Specifically, small-scale GI application provides a dispersed and passive treatment response to the spatially diffuse nature of stormwater runoff. This technology has comparable efficiencies to other traditional stormwater structures, with the added advantage of being incorporable into existing infrastructure (roadsides, private gardens, etc). Additionally, GI can be added as a technological focus within the South African WSUD framework.

Through a quantitative comparative investigation of removal efficiencies, small-scale dispersed GI elements (bio-filter grass strips, bio-filter grass swales and bio-retention structures) were compared to retention ponds, detention basins and wetlands in terms of median TSS and TDS mass removal efficiencies. It was found that, although retention ponds performed best overall for TSS median mass removals, the small-scale dispersed GI structures performed comparatively well (>0.75 median mass removals) and better than the detention basins and wetlands. TDS median mass removals were poor for all structures (~0.4), except for bio-filter grass strips (0.79) and retention ponds (0.93). However, all structures, including retention ponds, showed negative removals for both TSS and TDS, indicating that materials may be washed out of these structures during large storm events. The results therefore showed that the small-scale GI structures had comparable efficiencies to other stormwater structure types and generally performed better than detention basins and wetlands, making them a viable alternative.

The concept of GI is not unusual for application to the field of water treatment. The fundamental use of natural systems to treat water is not new and has manifested in the activated sludge system for sewage treatment, algal ponds for effluent treatment and reed beds for various applications. Although all of these technologies are not specifically “green”, they are natural and the leap in thinking towards using GI systems for stormwater treatment should not be great for the water-quality engineer. This technology is, however, not without its share of disadvantages and knowledge gaps. Much future research into many aspects, ranging from data collection to implementation, is warranted.

REFERENCES

ASCE (American Society of Civil Engineers) 2006. Standard guidelines for the design of urban stormwater systems. Reston, VA: ASCE.


TRCA (Toronto and Region Conservation Authority), CVC (Credit Valley Conservation Authority) 2010. *Low impact development stormwater management planning and design guide. Version 1*. Ontario, Canada: TRCA. Available at: [www.creditvalleyca.ca](http://www.creditvalleyca.ca) [accessed on 26 July 2017].


Concrete properties for ultra-thin continuously reinforced concrete pavements (UTCRCP)

A Mackellar, E Kearsley

Ultra-Thin Continuously Reinforced Concrete Pavement (UTCRCP) is an innovative road paving technology that can have significant advantages over traditional road paving techniques. Tests have shown that UTCRCP can carry in excess of one hundred million E80s (standard 80 kN axle loads). The laboratory tests used for quality control of conventional concrete are not adequate to fully capture the effects of the steel fibres added to the high strength concrete used in UTCRCP. In this study the concrete strength and fibre content were varied and the mechanical and physical properties of the concrete were measured. The tests included compressive strength, split cylinder, modulus of elasticity and four-point bending slab tests. The results of these tests are reported in this paper, and the suitability and shortcomings of the tests are discussed.

INTRODUCTION

An Ultra-Thin Continuously Reinforced Concrete Pavement (UTCRCP) as defined by Perrie and Rossman (2009) is a thin concrete layer with a thickness in the region of 60 mm constructed from concrete with a compressive strength in the order of 100 MPa that contains both steel and plastic fibres. Two types of ultra-thin pavements are defined by Perrie and Rossman (2009):

- UTCRCP for highly trafficked pavements
- UTCRCP for lower trafficked pavements.

Although these pavements share many features, they have some distinct differences. This study focused on UTCRCP intended for highly trafficked pavements.

With the ageing road network in South Africa and the limited budget available to perform maintenance of roads, there is a need to consider innovative and cost-effective rehabilitation options (Kannemeyer et al 2007). UTCRCP can be an option for the structural rehabilitation of roads. A number of experimental and trial sections have been constructed to test the performance of UTCRCP under vehicle loads, including accelerated pavement tests with the Heavy Vehicle Simulator (HVS) and trial sections on national highways.

The main components of UTCRCP are concrete, reinforcing mesh, steel fibres and polypropylene fibres. The high fibre content and concrete strength result in concrete that is relatively expensive compared to normal concrete. Higher-strength concrete can be less workable, while increased fibre content also reduces workability. Therefore increasing fibre content and concrete strength not only results in a product that is more expensive per unit volume, but the handling and placing difficulties also result in higher construction cost. Workability of the concrete can be increased by the appropriate use of concrete technology. However, this also has cost implications. If the fibre content and concrete strength can thus be reduced, this will result in a saving in the construction cost of UTCRCP. The risk, however, is that reducing concrete strength and fibre content will compromise the long-term performance of UTCRCP. An optimised UTCRCP design will balance construction cost with long-term performance. This study aimed to investigate the effects of varying the concrete strength and fibre content on the performance of UTCRCP.

UTCRP COMPONENTS

Concrete

The concrete compressive strength specified for construction is in the region of 90 MPa to 120 MPa. Laboratory tests have shown that the optimum thickness of UTCRCP is between 50 mm and 60 mm in order to
maximise bending resistance (Kannemeyer et al 2007). Reinforcing mesh needs to be placed at the centre of the UTCRPC. Placing concrete within these tight confines limits the maximum aggregate size that can be used in the concrete. The experimental sections constructed at Heidelberg used 6.75 mm stone (Kannemeyer et al 2007; Mukandila et al 2009). After placing, during curing and thereafter, concrete shrinks, and because this shrinkage is restrained the reduction in volume places the road pavement in tension. De Larrad (2005) observed that the concrete shrinkage can be used to the advantage of engineers. Placing the pavement in tension assists in restraining buckling of the pavement as a result of excessive thermal expansion.

Reinforcing mesh
It was found that increased diameter of the steel mesh resulted in increased bending resistance. However, thicker mesh also resulted in higher cost and increased the difficulty of placing the concrete. The mesh diameter recommended by Kannemeyer et al (2007) was 5.6 mm (Y6) with a spacing of 50 mm by 50 mm as a good compromise between cost and constructability. Shrinkage of the concrete can result in high-tensile forces in the reinforcing mesh, and this can be large enough to snap the reinforcing mesh (Perrie et al 2011). It is thus important to supply sufficient ductile mesh reinforcing.

Steel fibres
The inclusion of steel fibres can delay and control the tensile cracking of the concrete, thus increasing the load-carrying capacity of the concrete (Elsaigh et al 2005; Chen 2004). The use of the incorrect geometry fibres was found to lead to rapid failure of UTCRPC. The HVS and associated laboratory tests showed that 30 mm hooked-end fibres can prevent sudden failures (Kannemeyer et al 2007).

Polypropylene fibres
The function of polypropylene fibres added to concrete is different to that of steel fibres. Polypropylene fibres do not significantly contribute towards the flexural performance of concrete (Zhang & Stang 1998). The purpose of the polypropylene fibres is to modify the properties of the fresh concrete. In particular, these fibres have been found to reduce plastic shrinkage cracking of the concrete (Illston & Domone 2008).

**Experimental setup**
As the aim of this investigation was to determine the effect of both concrete strength and fibre content on the properties of the concrete, concrete mixes with target compressive strengths between 50 MPa and 90 MPa with steel fibre content up to 90 kg/m³ were used. Specimens were cast to determine and compare the 28-day compressive strength, split tensile strength, bending strength and stiffness of the different mixes.

**Concrete mix design**
The concrete mix designs are given in Table 1.

<table>
<thead>
<tr>
<th>Water-cement ratio</th>
<th>Cement (CEM I 52.5N)</th>
<th>Fly ash</th>
<th>Condensed silica fume</th>
<th>Water</th>
<th>Dolomite 9.5 mm aggregate</th>
<th>Dolomite sand</th>
<th>Superplasticiser</th>
<th>Defoamer</th>
<th>Polypropylene fibre</th>
<th>Steel fibre (30 mm long 0.5 mm diameter hooked-end cold-drawn wire fibres)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit</td>
<td>Relative density*</td>
<td>Desired concrete mean strength</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>90 MPa</td>
<td>70 MPa</td>
<td>50 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water-cement ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cement (CEM I 52.5N)</td>
<td></td>
<td>3.14</td>
<td>0.48</td>
<td>0.59</td>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fly ash</td>
<td></td>
<td>2.26</td>
<td>267.8</td>
<td>217.9</td>
<td>171.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Condensed silica fume</td>
<td></td>
<td>2.32</td>
<td>45.1</td>
<td>36.7</td>
<td>28.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td></td>
<td>1.00</td>
<td>22.5</td>
<td>18.3</td>
<td>14.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dolomite 9.5 mm aggregate</td>
<td></td>
<td>2.82</td>
<td>161.0</td>
<td>161.0</td>
<td>161.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dolomite sand</td>
<td></td>
<td>2.84</td>
<td>759.4</td>
<td>759.4</td>
<td>759.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Superplasticiser</td>
<td></td>
<td>1.05</td>
<td>1.268</td>
<td>1.333</td>
<td>1.393</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Defoamer</td>
<td></td>
<td>0.81</td>
<td>5.9</td>
<td>4.8</td>
<td>3.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polypropylene fibre</td>
<td></td>
<td>0.92</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel fibre (30 mm long 0.5 mm diameter hooked-end cold-drawn wire fibres)</td>
<td></td>
<td>7.68</td>
<td>0.50,70,90</td>
<td>0.50,70,90</td>
<td>0.50,70,90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Relative density measured by helium gas pycnometer

**Preparation of specimens**
Specimens (cubes, cylinders and slabs) were prepared in the laboratory and cured in water at 24°C for 28 days.

- The standard cube test (SANS 5863, 2006b) on 100 mm cubes to measure compressive strength. The results reported are the average of three test results.
- The modulus of elasticity or E-value test on three 150 mm diameter cylinders (300 mm high). A gauge length of 198 mm was used and the deformation for loads up to 40% of the cube strength was used to calculate the chord modulus of elasticity. The E-values reported are the average of three specimens tested. After the chord modulus was measured, specimens were loaded to 70% of the cube strength with deformation readings taken in 50 kN increments. This was done to examine the extent of linear elastic behaviour of the concrete.
- The split cylinder test (SANS 6253, 2006a) on three 150 mm diameter cylinders to measure indirect tensile strength.
- Flexural tests on three 700 mm × 250 mm × 55 mm slabs to measure

| Table 1 Concrete mix designs

<table>
<thead>
<tr>
<th>Water-cement ratio</th>
<th>Cement (CEM I 52.5N)</th>
<th>Fly ash</th>
<th>Condensed silica fume</th>
<th>Water</th>
<th>Dolomite 9.5 mm aggregate</th>
<th>Dolomite sand</th>
<th>Superplasticiser</th>
<th>Defoamer</th>
<th>Polypropylene fibre</th>
<th>Steel fibre (30 mm long 0.5 mm diameter hooked-end cold-drawn wire fibres)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit</td>
<td>Relative density*</td>
<td>Desired concrete mean strength</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>90 MPa</td>
<td>70 MPa</td>
<td>50 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water-cement ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cement (CEM I 52.5N)</td>
<td></td>
<td>3.14</td>
<td>0.48</td>
<td>0.59</td>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fly ash</td>
<td></td>
<td>2.26</td>
<td>267.8</td>
<td>217.9</td>
<td>171.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Condensed silica fume</td>
<td></td>
<td>2.32</td>
<td>45.1</td>
<td>36.7</td>
<td>28.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td></td>
<td>1.00</td>
<td>22.5</td>
<td>18.3</td>
<td>14.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dolomite 9.5 mm aggregate</td>
<td></td>
<td>2.82</td>
<td>161.0</td>
<td>161.0</td>
<td>161.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dolomite sand</td>
<td></td>
<td>2.84</td>
<td>759.4</td>
<td>759.4</td>
<td>759.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Superplasticiser</td>
<td></td>
<td>1.05</td>
<td>1.268</td>
<td>1.333</td>
<td>1.393</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Defoamer</td>
<td></td>
<td>0.81</td>
<td>5.9</td>
<td>4.8</td>
<td>3.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polypropylene fibre</td>
<td></td>
<td>0.92</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel fibre (30 mm long 0.5 mm diameter hooked-end cold-drawn wire fibres)</td>
<td></td>
<td>7.68</td>
<td>0.50,70,90</td>
<td>0.50,70,90</td>
<td>0.50,70,90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Modulus of Rupture (MOR), maximum load and energy absorption.

TEST RESULTS

Compressive strength
The effect of fibre content on compressive strength of concrete is shown in Figure 1. It is observed that fibre content has a minimal effect on compressive strength, even at high fibre contents. This concurs with the findings of Illston and Domone (2008), as well as Song and Hwang (2004). Thus the addition of steel fibres can be expected to have little effect on the compressive strength of concrete.

Although cube testing is a vital quality control test and should be used during the construction of UTCRCP, engineers must be aware that the compressive strength does not give any indication as to the quantity or effectiveness of the steel fibres in the mixture.

Modulus of elasticity
The chord modulus of elasticity and its relation to concrete strength is shown in Figure 2. Modulus of elasticity increases with increasing concrete compressive strength. While there is some difference in stiffness between specimens with differing fibre content, this difference appears not to be significant.

When taking multiple deformation readings during loading it was observed that the stress-strain response was not in fact linear. In order to distinguish between the standard test and the results obtained from taking additional readings, the term "stiffness" will be used for the stress-strain relationship obtained from the additional readings. Figure 3 shows the change in stiffness with increasing applied stress. It is observed that stiffness is not constant under increasing stress, and that fibre content has a small to negligible effect on stiffness.

In Figure 4 the stress applied to each specimen tested was normalised based on the ultimate compressive strength of the concrete for that mix. Additionally the stiffness was normalised based on the stiffness of the first loading increment. It can be observed that at 70% of the ultimate compressive strength, the stiffness reduces to 80% of the initial stiffness. It is also observed that, when the data points are normalised in this manner, all the results fall in a narrow group. This allows a curve to be fitted; the equation describing the observed behaviour is given in Figure 4. Understanding the nonlinear behaviour of concrete will be an advantage when finite
Element modelling is conducted. Nonlinear finite element models are available, and the nonlinear elastic behaviour of concrete can be incorporated into finite element models to enhance the accuracy of these models.

**Split cylinder strength and cracking stress**

The indirect tensile strength of the concrete was determined by using the split cylinder test. This is a standard test and the method is described in ASTM C496/C496M-04 (ASTM 2009a&b) and SANS 6253 (SANS 6253, 2006a). For the purpose of measuring the post-crack behaviour of the concrete, the horizontal deformation of specimens was recorded using Linear Variable Differential Transformers (LVDTs) attached to metal pins that were inserted into holes that had been drilled into the specimen. Two LVDTs were used, one at either end of the specimen, and the horizontal deformation was measured over a length of 50 mm. This method was described by Denneman et al (2011) and Denneman et al (2012).

The formula recommended by both the SANS (SANS 6253, 2006a) and ASTM (2009a) is derived from Boussinesq’s theory and was initially solved by Hertz (Timoshenko & Goodier 1951). The assumption made is that the load is applied as a line load. In practice, however, the load is applied over finite width. In order to take this effect into account Tang (1994) suggested the use of the modification indicated in Equation 1.

\[
f_t = \frac{2P}{\pi ID} \left[1 - \left(\frac{b}{D}\right)^2\right]^\frac{1}{2}
\]

(1)

Where:
- \(f_t\) is the tensile stress (MPa)
- \(P\) is the load (N)
- \(l\) is the length of the specimen (mm)
- \(D\) is the diameter (mm)
- \(b\) is the width of the load strip (mm).

The formula with the adjustment for load width has been successfully used by other researchers when numerically modelling the fracture behaviour of fibre-reinforced concrete (Denneman 2010). The result of a typical split cylinder test is shown in Figure 5. Values used are indicated in the figure as:

- **First Crack Stress:** this is taken as the stress at which the stress-strain relationship ceases to be linear (Johnston & Zemp 1991; ASTM 2004).
- **Principal Crack Stress:** the peak or plateau stress following shortly after the First Crack Stress.
- **Maximum Stress:** the maximum stress resisted by the specimen.

It should be noted that concrete with no fibres fails in a brittle manner. Thus the maximum stress will be equal to the principal crack stress.

The results in Figure 6 show a clear increase in principal cracking stress with increasing concrete compressive strength. The results also show that fibre content has little influence on the principal crack stress.
tensile strength of the concrete. This result was expected, because steel fibres can only carry significant tensile load after a crack has formed in the concrete. Fibres bridging the crack transfer load across the crack. Principal crack stress is governed by concrete strength and therefore fibre content does not play a significant role. While the split cylinder test gives a good measurement of the concrete tensile strength, it can be concluded that this test is not a good measure of the behaviour of the fibres in the concrete.

In order to measure the effect of the fibres, the post-crack behaviour of the specimen must be examined. Post-crack strength can be quantified using various methods. The comparison made here is to use the maximum stress results of the split cylinder test as shown in Figure 7. Similar to the principal crack strength results shown in Figure 6, there is an increase in tensile strength with increasing compressive strength; however, higher stresses are now exhibited for specimens with higher fibre contents.

Another method of evaluating post-crack performance of the split cylinders is to evaluate the strain energy. Strain energy is calculated as the area under the load deformation curve up to a specific deflection. This is equivalent to the work done up to that deformation. For the purposes of comparison, horizontal deformation limits of 0.25 mm and 1 mm were selected. These results are shown in Figures 8 and 9. It is clear from these graphs that at any strain level the fibres vastly improve the strain energy performance. At low strain levels there is little difference in the strain energy for different fibre contents; however, as the strain increases, the benefit of increased fibre content becomes apparent. There is an increase in strain energy with increased concrete strength. As reported by Abu-Lebdeh et al. (2011), this result can be expected as the improved bond between the fibres and concrete matrix increases the energy required to mobilise and pull out the fibres. For lower strains the effect of the concrete strength on strain energy is greater than that of fibre content. At higher strains this effect becomes less pronounced, to the point where increasing concrete strength no longer results in increased strain energy. It is at this point that increased fibre content starts to have more influence on strain energy than the concrete strength.

**Flexural slab tests**

Slab tests have an advantage over both cube and split cylinder tests because they give an indication of the performance of the pavement structure. Pavements are flexible and transfer force by bending. While it is not possible to replicate all the interactions a full pavement will be subjected to, slab testing attempts to measure the flexural capacity of UTCRCP by applying loads that cause bending of the slabs.

Flexural or beam tests are frequently used as an indirect measure of the tensile strength of concrete used in pavements. The standard test uses a concrete prism of 100 mm by 100 mm spanning 300 mm, loaded in 4-point bending (ASTM 2004). Using elastic theory the tensile strength of the concrete is determined. Denneman (2010) showed that fibre-reinforced concrete exhibits a strong size effect due to its high post-crack tensile capacity. The implication...
of this is that, in order to properly measure the performance of fibre-reinforced concrete, the laboratory specimen must be of the same depth as the UTCRCP. By testing 55 mm thick slabs (the thickness is the same as that of the constructed pavement) the size effect can be neglected, giving a better indication of the actual pavement performance.

Slab tests do not only provide information on the material properties of the concrete, but also the strength of the whole system, as the contribution of the reinforcing bars can also be taken into account.

Flexural tests were conducted on slab specimens that had the same thickness as the UTCRCP. Steel reinforcing mesh of 5.6 mm diameter high-yield steel with a spacing of 50 mm centre-to-centre was placed in the mid-height of the slab specimen. Slabs were supported to span 450 mm and loaded at third points in deflection control at a rate of 0.025 mm/min. From these results cracking strength, maximum load and energy absorption were determined.

The typical load deflection behaviour of a slab is shown in Figure 10. At the onset of loading the relationship between load and deflection is linear. This linear relationship continues up to the point where the principal crack occurs. This point is chiefly a measure of the tensile strength of the concrete. Without steel reinforcing or steel fibres, this would be the ultimate load carried by the sample. The stress in the outer fibres of the concrete, when the failure occurs, is known as the cracking strength or Modulus of Rupture (MOR). Thus the flexural strength of the concrete can be determined from the slab test.

The results of the slab MOR are shown in Figure 11. As expected, MOR is largely influenced by concrete strength, while fibre content has a marginal effect. The flexural strength of the concrete relative to its compressive strength can also be seen in this figure. Flexural strength of concrete is, as expected, approximately 10% of the compressive strength. The results do, however, show that this fraction decreases with increased compressive strength to less than 8%, indicating a diminishing benefit for flexural strength as compressive strength increases.

Energy absorption is a measure of the work done in order to deflect the specimen to a certain point. It is measured in Joules and calculated as the area under the load deflection curve to a specified deflection limit. The energy absorption values for slabs are presented in Figure 12. This
shows that energy absorption increases as deflection increases. Beyond 15 mm deflections the specimens broke and little further energy was absorbed. (Not all specimens exhibited the behaviour of the slab in Figure 10, breaking after approximately 8.5 mm; some continued to carry load beyond this point.)

In order to evaluate differences in energy absorption at low, medium and high deflections, three deflections were chosen – 1.8 mm for low deflection, 10 mm for mid-range deflection and 15 mm for large deflection. The energy absorption at these three deflections is plotted in Figure 13. At the low deflections the energy absorbed increases with increasing concrete strength. Increased fibre content also contributes to increased energy absorption (see Figure 13a). At high deflections this trend is reversed. Energy absorption decreases with increased concrete strength and fibre content (see Figure 13c). At mid-range deflections the energy absorption is approximately constant with increasing concrete strength (see Figure 13b).

The reason for this behaviour, i.e. the weaker concrete recording higher energy absorption at large deflections, could possibly be attributed to the concrete crushing in compression. The crushing of the compression concrete in the slab delays the failure of the reinforcing mesh as the strain is relieved. Thus, at larger deflections more energy can be absorbed. The mode of failure could be clearly observed during testing. Figure 14 shows a slab of 90 MPa concrete being loaded to high deflection. Cracking has advanced through the slab. The compression block at the top of the slab is very small. Figure 15 shows a specimen of 50 MPa concrete where cracking has not advanced through the slab. In the compression zone a horizontal crack is visible. This is an indication of the concrete crushing in compression.

CONCLUSIONS
The results of this study showed that the current quality control tests used for plain concrete need to be used with caution when applied to UTCSRCP. While the mechanical properties, such as concrete compressive and indirect tensile strength, can be measured, the current standard tests do not reflect the added advantages of the fibres on the performance of the road pavement.
In summary the results of this study showed that:

- The cube test can be used as a quality control test for concrete strength, but does not yield information on the quantity or effectiveness of the fibres.
- Modulus of elasticity increase with increasing concrete strength. Fibre content does not appear to have a significant influence. The compressive stress-strain relationship of concrete is not linear. When normalised, the stress-strain behaviour can be described by a single equation. This equation is applicable over a range of different concrete strengths.
- The standard split cylinder test can also be used to measure concrete tensile strength, but is a poor measure of the quantity and effectiveness of the fibres. However, if lateral displacement is measured during the test, the effect of the fibres can be measured.
- The flexural slab test is an effective quality control test in that it produces a number of performance measures. The slabs are cast with the same thickness as UTCRCP and also include reinforcing steel mesh. From the results of the slab tests, concrete strength, maximum load and energy absorption can be calculated. However, the results of this study show that these tests must be used with caution.
- Modulus of Rupture (MOR) is a good measure of concrete flexural strength, but is not a good measure of fibre content or performance. There is correlation between the flexural strengths measured from the slab tests and the splitting cylinder tests. This means that MOR could be used in place of split cylinder testing, streamlining on-site quality control tests.
- Maximum load is a poor indicator of performance. From these results it appears that it is not sensitive to concrete strength or fibre content. It is recommended that maximum load is not used as a quality control measure for UTCRCP.
- The energy absorption must be used with caution. At present the quality control specifications for energy absorption measured at large deflections favour the production of weaker concrete. It is recommended that the quality control specification should be adjusted for UTCRCP to energy absorbed at low deflections which are more appropriate to pavements.

REFERENCES


Influence of fibre content aspect ratio and type. 


The development of suitable cyclic loading and boundary conditions for ballast box tests

T C U Jideani, P J Gräbe

Laboratory tests on ballast give insight into the behaviour and performance of the ballast layer under passenger and heavy-haul traffic. It is important, however, to ensure that the simulation of train loads on the ballast layer in the laboratory represents in-situ loading conditions. Furthermore, the provision of ballast lateral confinement during laboratory tests should model the confinement along the track. With adequate, representative loading patterns and boundary conditions executed during laboratory tests on ballast, the overall response and performance of the ballast layer can be estimated and predicted more accurately. This gives an indication of an ideal response of the ballast layer in the field, as well as its impact on track structure deterioration.

The objective of this study was to develop suitable cyclic loading and boundary conditions for ballast box tests in the laboratory to represent similar conditions in the field. By conducting box tests, the ballast deformation results revealed the suitable loading pattern that produced a similar rate of ballast strain accumulation as the Field Loading (FL) pattern. Furthermore, boundary condition results showed that decreasing the Level of Lateral Confinement (LoLC) increased the permanent deformation of the ballast layer and the breakage of ballast. The laboratory loading pattern developed in this research, as well as comparable laboratory and field boundary conditions, could provide accurate predictions of the long-term behaviour of ballast and support the planning for subsequent ballast maintenance interventions based on realistic and accurate laboratory test results.

INTRODUCTION

Although ballastless track does exist, the conventional track to date is by far the most common and economical track utilised. The design of railway track is commonly based on axle load and wheel spacing. Reducing the wheel spacing causes the influence lines from each wheel load to overlap, causing higher stresses, thus increasing the vertical deflection of the track. Therefore, the length of a train and its wheel configuration are important factors to consider when determining the cumulative effect of cyclic loading on a track structure, especially when conducting laboratory test simulations.

Most laboratory box tests on ballast employ haversine loads to simulate train wheel loads. However, the effect of wheel spacing, loading impulse (the area under the load curve) and wheel load overlap is not considered when choosing a loading pattern. Furthermore, simulated train loads and boundary conditions as used in ballast box tests often do not correlate well with real train loading. It is therefore important to ensure that reasonable and practical conclusions are drawn based on realistic loading and boundary conditions employed in the laboratory.

LITERATURE REVIEW

Cyclic loading, from a railway point of view, is characterised by the shape, duration, magnitude of loading (stress) pulse, time interval between consecutive load pulses (a reflection of wheel spacing) and the total number of load pulses. Powrie et al (2007) and Li et al (2015) suggest that, to determine the number of cyclic loading applications for a laboratory test, common practice is to assume that two axle loads from a bogie are considered to produce one load cycle for the ballast layer, and four axles from two bogies are considered to produce a single load cycle for the subgrade layer. Selig and Waters (1994) presented a conversion factor to convert the number of load cycles to gross tonnage as follows:
\[ C_m = \frac{10^6}{A_Na} \] (1)

Where:
- \( C_m \) = the number of load cycles per million gross tonne (MGT)
- \( A_t \) = the axle load in tonnes
- \( N_a \) = the number of axles per load cycle.

However, the above practice and equation vary depending on the coupler/axle spacing of the train under consideration. Huang (1993) and Li (1994) suggest that the type and duration of cyclic loading should simulate the actual occurring loading pattern in the field, and recommended the use of a stress pulse in the form of haversine (Figure 1), triangular or trapezoidal loading. Most laboratory tests on ballast, either box or triaxial tests, employ haversine loads (comparable to pavement loading) to simulate train wheel loads (Indraratna & Ionescu 2000; Ebrahimi et al 2012). However, the effect of wheel spacing, loading impulse (area under loaded curve) and wheel load overlap is not considered when choosing a loading pattern. This loading condition for ballast laboratory testing needs to be evaluated and compared with multiple load pulses per load cycle which is identical to field loading conditions.

Field investigations using load cells (Zakeri & Sadeghi 2007; Sadeghi 2008; Sadeghi & Shoja 2012) and pressure cells (Gräbe et al 2005) reveal the loading pattern of a train (comprising 1800 mm wheel spacing per bogie) at the rail seat of a concrete sleeper and the changes in vertical dynamic stress pulses at each layer with depth, respectively (Figures 2 and 3(a)(b)).

Many field and numerical studies conclude that stress pulses decrease with depth – where each passing axle is apparent in the subballast layer, while the effect of each adjoining pair of bogies is evident on the surface of the natural ground. These studies further conclude that haversine loading is appropriate for subgrade tests in the laboratory (Li & Selig 1996; Liu & Xiao 2010; Priest et al 2010; Razouki & Schanz 2011; Li et al 2015).

The frequency of loading is dependent on the train speed, with typical track loading frequency between 8 Hz and 10 Hz (depending on the specified design speed of the railway line and the wheel spacing), according to Aursudkij et al (2009). Rest periods in field train loading is a function of axle wheel spacing and train speed, which have a major influence on ballast settlement (Qian et al 2011). Several studies conclude that continuous increase in loading frequency yields an increase in particle breakage, ballast axial and volumetric strains, stress and the deflection of individual track layers (Sun et al 2014; Yang et al 2009; Powrie & Priest 2011). Loading amplitude also has a major effect on ballast performance, where ballast permanent strain increases with an increase in wheel load (Stewart 1986; Selig & Waters 1994). However, the rate of ballast deformation is negligible at low cyclic stress levels, where the response of ballast below this stress level becomes almost elastic (Suiker 2002).

Although a continuous haversine load pulse is commonly applied in cyclic box and triaxial tests, the actual dynamic
loading may have a loading pattern of varying pulse shapes, with rest periods (according to the wheel spacing, wagon length, and train speed) which may directly affect the vibration and deformation of the track structure (Huang et al. 2009).

Permanent deformation of ballast in railway tracks is in the form of settlement, with the ballast layer being the main contributor to track settlement. Ballast settlement occurs due to particle rearrangement, abrasion, and particle breakage. Factors that affect ballast settlement (among others) include stress level, principal stress rotation, number of load applications, moisture content, stress history, density and load frequency (Knutson 1976; Selig & Waters 1994; Lekarp et al. 2000; Barksdale 1972; Shenton 1984; Lekarp & Dawson 1998; Indraratna et al. 2010a; Sun et al. 2014). Several ballast deformation models have been established based on the relationship between the number of load applications and settlement of ballast. However, these models do not consider sleeper properties, ballast type, ballast depth or shoulder dimensions (Abadi et al. 2016).

Ballast breakage is a common phenomenon that occurs during load application. Some factors influencing ballast breakage include macroscopic (external) stresses, size of the particle, coordination number (i.e. number of contacts with neighbouring particles), ballast properties (such as grain texture, mineral composition, internal bonding, etc) and loading conditions (McDowell et al. 1996; Hardin 1985; Lade et al. 1996; Shahin et al. 2007). Lade et al. (1996) summarised commonly used breakage indices which quantify the extent of ballast breakage. Marsal’s breakage index, $B_M$, is represented by the sum of positive values $\Delta W_k$ which are based on the difference in percentage retained on each sieve before and after a test. Indraratna’s Ballast Breakage Index (BBI), specifically for railway ballast, is used to quantify the extent of ballast degradation by evaluating the change in area of the particle size distribution before and after testing (Indraratna et al. 2005; Lackenby et al. 2007).

The influence of confining pressure on the behaviour of ballast is not considered a significant factor in rail track design. Figure 4 shows the effect of confining pressure on particle degradation using the ballast breakage index (BBI). Indraratna et al. (2005) and Lackenby et al. (2007) divide the ballast degradation behaviour into three zones (see Figure 4) following a series of

![Figure 3](image-url) (a) Position of in-situ pressure cells and (b) vertical stress plots for each layer of the track substructure (modified from Gräbe et al. 2005)

![Figure 4](image-url) Effect of confining pressure on particle breakage, with degradation zones (Indraratna et al. 2005; Lackenby et al. 2007)
triaxial tests. These zones are (I) Dilatant Unstable Degradation Zone (DUDZ), (II) Optimum Degradation Zone (ODZ), and (III) Compressive Stable Degradation Zone (CSDZ). Most ballast degradation is due to the breakage of angular corners in DUDZ, attrition of asperities in ODZ and particle splitting due to microcracks, particle flaws and fatigue in CSDZ. Furthermore, ballast axial strain decreases with increasing confining pressure.

Although researchers have performed numerous tests using the box test technique, the effect of different loading patterns and load impulse on the rate of ballast degradation is not fully understood. In addition, assessing the effect of confining pressure on ballast breakage from a ballast box test perspective under field loading and boundary conditions has not yet been investigated. This study therefore, seeks to provide a suitable loading pattern and boundary condition for laboratory ballast box testing to accurately reflect field conditions. To achieve these objectives:

- A suitable loading pattern was developed to reproduce an approximate rate of ballast strain accumulation as experienced in the field. This was done by comparing different haversine loading patterns with an FL (Field Loading) pattern.
- The effect of varying LoLC (Level of Lateral Confinement) on the permanent strain and particle breakage of the ballast layer was investigated.

**Table 1** Comparison of track details from field experiments

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Track gauge (mm)</td>
<td>1 065</td>
<td>1 435</td>
</tr>
<tr>
<td>Rails (kg/m)</td>
<td>60</td>
<td>UIC60</td>
</tr>
<tr>
<td>Sleeper type</td>
<td>PY concrete</td>
<td>B70 pre-stressed concrete</td>
</tr>
<tr>
<td>Sleeper spacing (mm)</td>
<td>650</td>
<td>760</td>
</tr>
<tr>
<td>Ballast thickness (mm)</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Subballast thickness (mm)</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>Passing speed (km/h)</td>
<td>40–50</td>
<td>40</td>
</tr>
<tr>
<td>Axle load (ton/axle)</td>
<td>20/26</td>
<td>22.5</td>
</tr>
<tr>
<td>Wheel spacing (mm)</td>
<td>1 830 (four-wheel load configuration)</td>
<td>1 800</td>
</tr>
</tbody>
</table>

**Table 2** Summary of variables used to calculate the dynamic wheel load and maximum rail seat load ($q_r$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axle load (heavy-haul)</td>
<td>tonnes</td>
<td>26</td>
</tr>
<tr>
<td>Static wheel load, $P_{\text{static}}$</td>
<td>kN</td>
<td>127.53</td>
</tr>
<tr>
<td>Vehicle speed, $V$</td>
<td>km/h</td>
<td>70</td>
</tr>
<tr>
<td>$\eta$</td>
<td></td>
<td>1.07</td>
</tr>
<tr>
<td>Track condition, $\delta$</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>$t$</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Dynamic impact factor, $\varphi$</td>
<td></td>
<td>1.429</td>
</tr>
<tr>
<td>Dynamic wheel load, $P_{\text{dyn}}$</td>
<td>kN</td>
<td>182.2</td>
</tr>
<tr>
<td>Sleeper spacing</td>
<td>mm</td>
<td>650</td>
</tr>
<tr>
<td>Load distribution factor</td>
<td>%</td>
<td>50.4</td>
</tr>
<tr>
<td>Maximum rail seat load, $q_r$</td>
<td>kN</td>
<td>92</td>
</tr>
</tbody>
</table>

**Figure 5** Loading pattern on rail seat from train axle loads experienced by a sleeper

**CHARACTERISING AND MODELLING OF THE FIELD LOADING PATTERN**

A typical FL pulse (pattern) at the rail seat, expressed as a load-time relationship, was recorded by Sadeghi (2008) with a wheel spacing of 1 800 mm (Figure 2). Comparing the shape of this load pulse with the stress pulse obtained from in-situ pressure plate readings at the subballast layer, recorded by Gräbe et al (2005) (Figure 3(b)) with a wagon wheel spacing of 1 830 mm, reveals some similarities in the pulse shapes – especially in the upper region. Other track and train details from the above-mentioned publications are outlined in Table 1.

The stress pulse shape recorded by Gräbe et al (2005) was used as a reference, because the focus of this study was on replicating the loading pulse of a 4-axle train configuration (South African Coal Line) in laboratory box tests. Hence, the FL pulse used for the experiment takes the shape of the stress pulse at the subballast (Figure 3(b)). Although wheel load data from strain gauges was readily available, the load on the ballast was assumed to be equivalent to the rail seat load due to the load distribution. Therefore, for all
experiments conducted, a maximum load equal to the rail seat load was used.

To convert the stress pulse values to rail seat loads, the equivalent force, \( f_n \), from the in-situ pressure plate, was calculated by multiplying the pressure plate readings, \( \sigma_n \), by the circular area of the pressure plate, \( a \) (Equation 2).

\[
f_n = \sigma_n \times a
\]  

Where: \( n \) is the data point number and the pressure plate radius is 30 cm.

To obtain the FL pattern (load-time relationship), the ratio of maximum rail seat load (\( q_r \)) to maximum equivalent force (\( max(f_n) \)) was multiplied by each \( f_n \) value. The maximum rail seat load (\( q_r \)) was obtained by multiplying the dynamic wheel load (\( P_{dyn} \)) by a load distribution factor of 0.5 (Sadeghi 2008). The dynamic wheel load (\( P_{dyn} \)) was calculated using the dynamic impact factor developed by Eisenmann (1972). Table 2 provides a summary of the dynamic wheel load and maximum rail seat load (\( q_r \)) calculation, with a maximum rail seat load of 92 kN.

The FL pattern (Figure 5) was simplified to eliminate irregularities. The following aspects of the loading pattern were adjusted:

- Wheel load magnitudes (peaks): an equal maximum wheel load of 92 kN was set for all four-wheel loads represented by the loading pattern.
- Averaged load valleys between wheel loads were equal.
- Wheel spacing: equal wheel spacing for a load cycle was assumed, with a negligible effect on the ballast material tested.

The average load valley was calculated as 55.2 kN. A relationship between load amplitude and load valley was established using a factor of 1.667. Therefore, for any minimum applied load (\( L_{min} \)) and load amplitude (\( L_{ampl} \)), the load valleys (\( L_{valleys} \)) between wheel load can be obtained using the following expression:

\[
L_{valleys} = L_{min} + \frac{L_{ampl}}{1.667}
\]  

The ability of the hydraulic load frame to simulate this loading pattern was investigated by obtaining a suitable frequency and load amplitude to produce an appropriate loading shape. This was performed using the sleeper block and a 500 mm × 500 mm × 350 mm steel box containing ballast (300 mm depth). Consequently, a scaled-down load was determined. The Axial Command values in the MTS load profile were adjusted to reduce the load amplitude required to achieve the desired simplified FL pattern represented by the Axial Force values as shown in Figure 6. A scaled-down (rail seat) load amplitude, and a minimum and maximum load of 10 kN, 5 kN and 15 kN respectively, at 2.5 Hz, were obtained after the Axial Command values were adjusted.

**EXPERIMENTAL SETUPS AND METHODOLOGY**

Two experimental setups (1 and 2) were considered. The experiment using Setup 1 was concerned with developing a suitable loading pattern for laboratory box tests with the boundary conditions remaining constant (fully confined). The experiment using Setup 2 was concerned with altering the boundary conditions (decreasing the lateral confinement) of a box test to investigate the effects on ballast settlement and breakage.

**Experimental Setup 1 – simulation of loading patterns**

This scaled-down model of a railway track consisted of a 25 mm thick steel box (500 mm × 500 mm × 350 mm) and a PY sleeper block (228 × 185 × 170 mm). The setup required regular ballast replacement, as four different loading patterns were investigated. Figures 7(a) and (b) show the schematic drawings of the ballast compaction and experimental arrangement for Setup 1 respectively.

The quartzite ballast used complied with the S406 ballast specification (TFR 1998) for grading. The 450 mm × 440 mm rigid compaction frame was constructed.
to ensure limited flexural movement. The rubber paddings placed between the actuator piston and the rigid frame, and between the ballast and steel sheet, were used to prevent damage abrasion and reduce ballast abrasion during compaction, respectively. Track subgrade is commonly modelled as a semi-infinite elastic half space, characterised by a soft or stiff foundation. The rubber sheets at the base of the box were used to replicate the slightly compressible effects of the subgrade layer. However, the contribution of the rubber sheets to achieve this subgrade provided by the load unit table in a box setup is always a limitation compared to the proper use of a semi-infinite elastic half-space. After compaction, a high-density polyethylene (HDPE) Pandrol pad was fitted to the base of the sleeper block to prevent progressive sleeper wear. The axial force applied to the ballast sample was generated using a 500 kN hydraulic actuator (MTS component) regulated by servo-valves.

Methodology (Setup 1)
The comparison of alternative loading patterns to the FL pattern was primarily based on the rate of strain accumulation of the ballast layer. Other loading aspects that were investigated were the load cycle impulse and load frequency (rest periods). The FL pattern was compared with the following loading patterns which can be simulated in the laboratory:

- Conventional haversine loading (or Laboratory Loading – Lab L): frequently used in the laboratory to simulate train loading, where a single load cycle represents four-wheel loads.
- Impulse Haversine Loading (IHL): modified Lab L, where the impulse of the Lab L pattern is equal to that of the FL pattern, and a single load cycle represents four-wheel loads.
- Haversine Loading (HL): four loading pulses which represent four-wheel loads, with the load cycle impulse equal to the load cycle of the FL pattern.

Adjusted Haversine Loading (AHL): a modified version of the HL pattern, where the load amplitude was reduced to closely match the ballast deformation caused by the FL pattern.

Table 3 summarises the methodology for Setup 1. Each ballast sample was compacted in three layers following the rodding procedure outlined in the ASTM C29 standard (ASTM 2017). Further compaction was conducted in stages, yielding a total of 25 000 haversine cyclic loads. A preload of 1 kN was applied, with a compaction load of 85 kN – which was slightly greater than the calculated traffic load to prevent excess ballast deformation while simulating train operations. All samples were compacted to a void ratio of 43% with a target ballast mass of 112.5 kg. Data was captured at 100 Hz for all compaction stages. Table 4 compares each alternative loading pattern with the FL pattern, based on selected load properties. Graphic illustrations of a load cycle for each loading pattern are shown in Figure 8. Load applications (or load pulses), which represent

Table 3 Test procedure for Setup 1

<table>
<thead>
<tr>
<th>Sample</th>
<th>Cyclic compaction</th>
<th>Loading pattern</th>
<th>Rest period (FL only)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stage 1</td>
<td>Stage 2</td>
<td>Stage 3</td>
</tr>
<tr>
<td>A</td>
<td>40 kN at 5 Hz (5 000 cycles)</td>
<td>40 kN at 10 Hz (5 000 cycles)</td>
<td>85 kN at 10 Hz (15 000 cycles)</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Alternative loading patterns compared with field loading based on selected load properties

<table>
<thead>
<tr>
<th>Loading pattern</th>
<th>Equal impulse</th>
<th>Equal frequency</th>
<th>Equal load amplitude</th>
<th>Load pulses per cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lab L</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td>IHL</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td>HL</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>4</td>
</tr>
<tr>
<td>AHL</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 8 Shapes of the different loading patterns
the influence of wheel loads, are contained in a load cycle. Hence, the number of load applications vary for each loading pattern depending on their characteristics. After compaction, 500 000 load cycles were applied to each sample, with 100 000 load cycles applied in each stage (Stages 1 to 5) during the testing of the loading patterns (Table 4). The effect of rest periods (bogie spacing and time interval between trains) on the permanent settlement of the ballast layer was only assessed on Sample B using the FL pattern at 2.5 Hz. The number of cycles for each loading pattern (based on the duration rest period) was kept constant at 300 000 cycles.

**Experimental Setup 2 – influence of confinement**

This setup involved a full-scale box test with an adjustable length to investigate the effect of lateral confinement on ballast settlement and breakage. The steel box, 1 630 mm long, 650 mm wide and 450 mm deep, was built to represent a half sleeper bay of a typical railway track, considering track longitudinal symmetry and the minimum requirements of the ballast layer for a 1 065 mm gauge South African Coal Line (TFR 2012). A uniform stress distribution was assumed to occur at the base of the half sleeper (Talbot 1993). Schematic illustrations of the materials, measuring devices, and equipment and their locations, are shown in Figure 9. A mild steel bar (377 mm × 150 mm × 150 mm) was used to link the sleeper to the hydraulic actuator piston. The mild steel bar was milled to form a bevelled edge (1 in 20 slope) to level off the rail-seat slope on the sleeper. Two mild steel bars (230 mm × 35 mm × 35 mm) were used to support 20 mm steel rods passing through the fastener slots on the sleeper. Two external HBM WA/50 mm L-plungers were connected to the MTS controller, which recorded the analogue signal of the sleeper displacement.

![Figure 9 Schematic (cross section (a) and confinement (b)) illustration of experimental materials and instrumentation for Setup 2](image-url)
Methodology (Setup 2)
The experimental procedures for setup were employed to quantify the breakage of ballast and to measure the settlement of the ballast layer while varying the Level of Lateral Confinement (LoLC). The term LoLC refers to the percentage of the ballast shoulder that is being confined in the lateral direction in the ballast box. The large steel box was designed to accommodate five levels of lateral confinement, namely 100% (fully confined), 75%, 50%, 25% and 0% (full ballast shoulder), as shown in Figure 9.

Fresh ballast samples were used for each LoLC. Ballast breakage was assessed at two locations in the ballast layer – at 100 mm and at 300 mm from the base of the box. Ten kilograms of ballast were painted sparingly with red and gold paint at 100 mm and 300 mm depths respectively, for each sample, to retain the surface friction of the ballast. The painted ballast was placed unconfined in the region below the sleeper loaded area, with the painted ballast at 300 mm, overlaid with normal ballast to achieve ballast breakage due to interparticle contact forces. Each sample was compacted in three layers following the rodding procedure outlined in the ASTM C29 (ASTM 2017) standard. Further compaction was conducted by applying 5 000 and 20 000 cyclic loads of 45 kN and 90 kN respectively, at 10 Hz using the MTS hydraulic actuator.

For a 75%, 50%, 25% and 0% lateral confinement, a 2.5 mm steel sheet (630 mm x 460 mm) was placed against the gate prior to compaction to maintain the compacted state of the ballast layer at 100% confinement. After compaction, the gate was removed and placed at the required confinement slot as shown in Figure 10(a). Figure 10(b) shows the steel sheet separating the compacted ballast and the ballast shoulder for 0% lateral confinement. The crib ballast was placed around the sleeper and rod compacted. The AHL pattern was applied at 10 Hz with a load range from 2.5 to 91 kN. The number of wheel load applications was 1 300 000 cycles (325 000 load cycles). Sieve analyses were conducted on the painted ballast.

Table 5 Summary of ballast sample response after compaction

<table>
<thead>
<tr>
<th>Sample</th>
<th>Settlement (mm)</th>
<th>Stiffness (kPa/mm)</th>
<th>Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16.5</td>
<td>108.0</td>
<td>32.4</td>
</tr>
<tr>
<td>B</td>
<td>15.6</td>
<td>108.8</td>
<td>32.6</td>
</tr>
<tr>
<td>C</td>
<td>18.2</td>
<td>168.7</td>
<td>50.6</td>
</tr>
<tr>
<td>D</td>
<td>17.2</td>
<td>154.7</td>
<td>46.4</td>
</tr>
</tbody>
</table>

Figure 11 Permanent strain accumulation for each sample during alternating loading patterns
ballast at 100 mm and 300 mm at the beginning and end of the cyclic loading.

RESULTS AND DISCUSSION

Ballast compaction
A summary of the ballast response for each sample after compaction is shown in Table 5. Possible factors affecting the changes in ballast response include varying particle shape and sizes and particle arrangement, among others. It is clear from the results that, even though the compaction resulted in similar total settlement, the final stiffness values differed too much to regard the samples as identical for comparative testing, hence the loading procedure in Table 3.

Axial strain of ballast layer under different loading patterns
Figure 11 shows the percentage strain of each ballast sample subjected to alternating loading patterns of FL and the other loading patterns. Samples A and B exhibited similar strain behaviour comprising a significant increase in the rate of strain accumulation after 400 000 cycles. Possible reasons for this observation are:

- Both the Lab L and IHL patterns do not have the same quantity of load pulses per cycle when compared with the FL pattern.
- Particle rearrangement or ballast breakage.

Considering the degradation zones established by Indraratna et al (2005) (Figure 4), the zone that best describes the deformation behaviour of each ballast sample is the compressive stable degradation zone (CSDZ). In this zone, the particle movements are restricted with a high coordination number due to the high level of confinement.

To compare the rates of strain accumulation between the FL pattern and each alternative loading pattern after 100 000 cycles, linear relationships between the percentage axial strain and traffic in million gross tonnes (MGT) were expressed. The number of cycles can be expressed in terms of MGT using Equation 1. Plots of percentage axial strain against traffic for every 10.4 MGT were created to observe the strain accumulation rate between the FL and alternative loading patterns in Figure 12.

Figure 12 Axial strain accumulation of ballast layer (samples) for every 10.4 MGT – (a) FL and Lab L, (b) FL and IHL, (c) FL and HL, and (d) FL and AHL
Two pairs of axial strains were compared for each plot, namely: Lab L 1 – FL 2 and Lab L 2 – FL 3 as shown in Figure 12(a). Linear regression lines were fitted to obtain the rate of strain accumulation (denoted as $m$ – percentage strain/MGT). For plots that indicate possible particle breakage or rearrangement (such as plot FL 3 after 4 MGT – Figure 12(a)), the rate of strain accumulation for the gradual portion, after breakage, was computed.

Sample A – field loading (FL) and laboratory loading (Lab L) patterns
Common properties of the FL and Lab L patterns include load amplitude and frequency. Dissimilar properties of these loading patterns include impulse and number of load pulses per load cycle (Table 4). The Lab L pattern is a generic sinusoidal waveform used to simulate train loading on ballast in the laboratory.

Figure 12(a) shows the plots of percentage axial strains of the ballast layer induced by the FL and Lab L patterns. It is expected that the rate of strain accumulation would decrease with traffic (number of cycles) for ideal situations, except in the event of weak spots in a track structure. However, there is a significant increase in strain rate after the transition from Lab L to FL for both pairs of axial strain, as FL yields a higher plastic strain of the ballast layer than Lab L. It is evident that the Lab L pattern is not capable of reproducing the strain under field conditions on the ballast layer. A major aspect of the loading pattern influencing the rate of plastic strain is the number of load pulses (or load applications). In this study, the number of load pulses refers to the number of load peaks within a load cycle as illustrated in Figure 5. Therefore, for a single load cycle, the FL pattern has four load pulses, while the Lab L pattern has one load pulse (which can also be referred to as one load cycle). Therefore, the amount of plastic strain produced per FL load cycle is significantly more than the amount of plastic strain produced per Lab L cycle.

Sample B – field loading (FL) and impulse haversine loading (IHL) patterns
Common properties of the FL and IHL patterns include load amplitude and load impulse. Dissimilar properties of these loading patterns include frequency and number of load pulses per load cycle. IHL is a modified loading pattern from the Lab L pattern, where the load impulse of the Lab L was increased to match the FL impulse per cycle.

Figure 12(b) shows the percentage axial strain plots of the ballast layer generated by the FL and IHL patterns. The FL pattern applied more load pulses than the IHL pattern; therefore the ballast layer deformed more under the FL pattern. Similar ballast strain behaviour is observed with Sample A (with the FL and Lab L patterns – Figure 12(a)). However, comparing the plastic strains induced by the IHL and Lab L patterns, there is a notable increase in the rates of plastic strain due to an increase in load impulse.

Sample C – field loading (FL) and haversine loading (HL) patterns
Common properties of the FL and HL patterns include load amplitude, load impulse and number of load pulses per cycle. A dissimilar property of these loading patterns is the load frequency. The HL pattern consists of four load pulses per cycle. Each load pulse has equal load amplitude, as in the case of the FL pattern.

Figure 12(c) shows percentage axial strain plots of the ballast layer generated by the FL and HL patterns. The HL pattern in the first pair caused a higher strain rate than the FL pattern. However, in the second pair, the strain rate caused by the FL pattern was higher than the HL pattern, due to possible particle breakage or rearrangement. The frequency and load amplitude of intermediate load pulses per cycle differentiate these two loading patterns. It is important to note, however, that in a perfect scenario (i.e. absence of possible ballast breakage or rearrangement), the strain rate caused by the HL pattern marginally exceeds the strain rate caused by the FL pattern for the second pair. The driving factor for this behaviour is the effect of the additional load amplitude provided by the second and third load pulse in the HL pattern. This increases the stress level per cycle exhibited by the HL pattern, which generates more strain deformation than the FL pattern. Similar strain responses with these loading patterns were observed in preliminary tests.

Sample D – field loading (FL) and adjusted haversine loading (AHL) patterns
Common properties of the FL and AHL patterns include load impulse and number of load pulses per cycle. Dissimilar properties of these loading patterns include load amplitude and frequency. The AHL pattern consists of four load pulses per cycle. After conducting several preliminary tests with varying load amplitudes of the AHL pattern, a final load amplitude of 8.5 kN was employed. The frequency of this loading pattern was determined based on the changes to the load amplitude and impulse.

Figure 12(d) shows the plots of percentage axial strain of the ballast layer generated by the FL and AHL patterns. Both pairs of plots have slight differences in the rates of axial strain, with a maximum strain difference of 2.2% per MGT. The absence of a sudden increase in axial strain is possibly due to the absence of ballast breakage or...
rearrangement, and equal amounts of stress levels applied to the test sample which eliminate potential rearrangement or breakage of ballast particles. All the axial strain plots were zeroed to quantify the extent of axial strain after every 10 MGT. For each axial strain plot pair of the FL and alternative loading pattern, the difference in strain accumulation was computed. Figure 13 shows the percentage axial strain difference between the FL and alternative loading pattern after every 10 MGT. The AHL pattern had the lowest strain difference, while the Lab L had the highest strain difference in relation to the axial strains induced by the FL pattern. Although these results were obtained with a rigid bottom box, the settlement trend across the different loading patterns will be similar under ideal/actual track conditions. Under ideal track conditions, however, the magnitude of settlement will be greater, as there will be settlement of the underlaying layers, unlike the rigid bottom used during the laboratory tests.

**Effect of load impulse on plastic strain of ballast layer**

Increasing the loaded area of the Lab L pattern to match the FL pattern involved reducing the load frequency of a single Lab L cycle while maintaining a constant load amplitude. The comparison of the strain rates resulting from these loading patterns (Lab L and IHL) is valid, as the initial ballast layer response of each test sample after compaction was relatively similar (Table 5). Referring to Figure 13, the effect of load impulse is clearly observed from the decrease in strain difference between FL-Lab L 1 and FL-IHL 1.

**Effect of rest periods on ballast permanent settlement**

Figure 14 shows the ballast permanent deformation caused by different loading patterns characterised by varying rest period intervals. Low permanent settlements were obtained due to the nature of the subgrade (rigid bottom of the box — possibly stiff, rocky formation in practice), high levels of confinement and ballast compaction. In practice, ballast settlement can be measured using multi-depth deflectometers placed at each layer below the ballast layer.

For Stage Rest Period 1 (Stage RP 1), taking the settlement caused by an FL pattern with no rest period as a reference, there was an 83% and 86% reduction in settlement for the FL pattern of 0.4 s and 0.8 s respectively. For Stage Rest Period 2 (Stage RP 2), taking the settlement caused by an FL pattern with 0.8 s rest period as a reference, there was a 10% reduction in settlement caused by the FL pattern with 0.4 s rest period, while a 110% increase in settlement caused by an FL pattern without rest period was recorded.

In summary, increasing the number of load pulses per given time increases the rate of ballast permanent settlement.

**Boundary conditions**

**Permanent strain of ballast layer**

Figure 15 shows the axial strain deformation of the ballast layer at each LoLC with a constant deviatoric applied stress across all

---

*Figure 14 Final settlement of the ballast layer after increasing and decreasing intervals of rest periods*

*Figure 15 Axial strain deformation of the ballast layer at each LoLC*
samples. The term “0%” confinement refers to the percentage of the ballast shoulder that is confined in the lateral direction by the steel gate of the ballast box. It is important to note that the number of cycles (Figure 15) is represented as a quarter of the number of load pulses (which were 1.3 million) due to a four-wheel configuration. This adjustment to the number of cycles agrees with the settlement trends of the ballast layer, as it is expected that, at a million cycles, the settlement trend would have levelled off significantly where gradual degradation occurs with time, as seen from previous research conducted on ballast.

Discrepancies in Figure 15, such as irregular axial strain plot of the ballast layer at 0% confinement and similar final ballast strain for 25% and 50% confinements, could be due to slow deformation response after compaction and uncontrollable variations in ballast layer response (such as stiffness, modulus, etc) respectively.

Ballast degradation behaviour under cyclic loading for the various levels of lateral confinement can be classified into the degradation zones established by Indraratna et al (2005) following the explanations below:

- Ballast degradation behaviour at 100% confinement can be classified within the compressive stable degradation zone (CSDZ) where a high lateral confinement significantly restricts particle movement, reorientation and rearrangement.
- Reducing the lateral confinement to 75% increased the final ballast settlement by 19.3 mm – a 70% increase in ballast settlement. This led to increased deflection and permanent settlement per load pulse.
- For 50% lateral confinement, the ballast layer had a permanent settlement of 57.6 mm – a 10.7 mm settlement difference (22.8% increase in vertical settlement) compared to the 75% confinement case. The permanent settlement of the ballast layer at 25% and 50% lateral confinement was similar at approximately 57 mm. Ballast degradation at 50% and 25% lateral confinement can be described by the particle behaviour and degradation that occur in the optimum degradation zone (ODZ).
- High levels of ballast settlement were observed and recorded for the ballast layer at 0% confinement, with a vertical settlement of 68.9 mm. The degradation zone that best describes the behaviour of particles under cyclic loading at 0% confinement is the dilatant unstable degradation zone (DUDZ). Ballast particles in this zone are subjected to low confining lateral pressures under high deviatoric stresses which settle significantly with time. Particles had more room to displace, as the LoLC decreases under the influence of constant loading, leading to increased axial deformation and particle breakage.

![Figure 16 Linear (a) and quadratic (b) relationships between final ballast settlement and LoLC](image)

Figure 16 shows linear and quadratic relationships between final permanent settlement and LoLC after 325 000 cycles (1.3 million load pulses). These relationships clearly indicate a general increase in ballast layer settlement with decreasing lateral confinement from 100% to 0% by 150%. A quadratic relationship has a better fit than a linear relationship.
Consider the relationship between BBI and the effective confining pressure developed by Indraratna et al. (2005) (Figure 4), the quadratic relationship between the permanent settlement and LoLC matches the BBI at lower vertical applied stress \((q_{max} < 230 \text{ kPa})\).

### Ballast Breakage

The difference in mass before and after cyclic loading for each lateral confinement at 100 mm and 300 mm were compared as suggested by Abadi et al. (2016) (Figure 17). This shows a clear indication that ballast breakage is significant closer to the sleeper, as well as for a ballast layer supported by a ballast shoulder without additional lateral confinement.

As expected, there was more particle degradation at 300 mm (particles closer to the sleeper) at 100% and 0% lateral confinement than at 100 mm (Table 6). However, for 75% to 25% lateral confinement there was more particle breakage for particles at 100 mm than at 300 mm, contrary to expectation.

Possible contributing factors to the discrepancies in BBI values between levels of confinement could be:
- Displaced painted ballast particles that are not within the sleeper loading region due to particle rearrangement during compaction. This movement is assumed to be more significant at 300 mm than at 100 mm, as high inter-particle contact forces decrease with depth (Han 2012).
- The probability of the same particle passing through the appropriate sieve during sieve analysis after testing might be low in some instances. Furthermore, Abadi et al. (2016) stated that the degree of particle breakage was not sufficiently detected by comparing PSDs before and after cyclic loading test.
- The number of ballast particles (overall sample size) may not be statistically significant and might have contributed to the discrepancies in these results.

The most common forms of ballast breakage, among others, are attrition of asperities and corner breakage. Other forms of breakage are particle splitting along weak planes, and particle fatigue. A major governing factor stimulating ballast breakage is the fracture strength of individual ballast particles. Therefore, to ensure limited degradation of the ballast layer and track structure, obtaining high quality ballast particles on the railway track (especially heavy-haul lines) is necessary.

### CONCLUSIONS

Laboratory ballast box tests were conducted on quartzite ballast to develop a suitable cyclic loading pattern which produces similar deformation rates as experienced in the field. The effects of varying lateral confinement on ballast layer strain and ballast breakage were also investigated. The following conclusions can be drawn from this study after the analysis and discussion of results:

- The FL pattern had more load pulses than the Lab L patterns, which contributed to increased rates of axial deformation. Employing the FL pattern thereafter caused a significant increase in the strain rate due to possible ballast rearrangement or breakage under more load pulses per cycle.
- Although the Lab L and IHL patterns have a single load pulse, there is a notable increase in the rate of axial strain of the IHL pattern compared to that of the Lab L pattern. This increase in strain rate is due to an increase in the impulse (loaded area) of the Lab L pattern as simulated by the IHL pattern.
- The rate of strain accumulation of the HL pattern surpasses the FL strain rate marginally because of the increased loading effects of the second and third load pulses in comparison to their corresponding load pulses in the FL pattern.
- By modifying different aspects of the HL pattern, the AHL pattern was developed. By decreasing the load amplitude of the HL pattern, this loading pattern (AHL) reproduced similar rates of strain compared to the FL pattern considered in this study. The AHL pattern is the most appropriate and suitable loading pattern to be implemented in laboratory tests when considering axial ballast strain. Employing this loading pattern will produce accurate and realistic results to predict ballast layer behaviour and track structure response during field loading.
- The permanent settlement of the ballast layer decreased with an increase in the rest period interval between load cycles, and increased with a decrease in the rest period interval.
- Ballast settlement increased by 150% when the lateral confinement in the

### Table 6 Ballast Breakage Index (BBI) at 100 mm and 300 mm for each LoLC

<table>
<thead>
<tr>
<th>Level of confinement (%)</th>
<th>BBI At 100 mm</th>
<th>BBI At 300 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.0019</td>
<td>0.034</td>
</tr>
<tr>
<td>75</td>
<td>0.166</td>
<td>0.073</td>
</tr>
<tr>
<td>50</td>
<td>0.084</td>
<td>0.048</td>
</tr>
<tr>
<td>25</td>
<td>0.097</td>
<td>-0.023</td>
</tr>
<tr>
<td>0</td>
<td>0.086</td>
<td>0.251</td>
</tr>
</tbody>
</table>

**Figure 17** Difference in mass before and after cyclic loading showing the effect of lateral confinement on changes to the total mass of ballast at 100 and 300 mm before and after cyclic loading.
ballast box tests was reduced from 100% to 0%.
- A fully confined ballast layer prohibited particle breakage to a large extent, while significant ballast breakage occurred in ballast layers with a shoulder profile.
- Ballast breakage indices were useful in quantifying the degree of ballast breakage by identifying changes at sieve size levels. Alternatively, obtaining the difference in total mass before and after cyclic loading provided a broad perspective on the degree of ballast breakage.
- Obtaining high quality ballast particles on the railway track (especially heavy-haul lines) is necessary to limit track structure degradation through ballast breakage.

**ACKNOWLEDGEMENTS**

The authors would like to acknowledge and thank Transnet Freight Rail for their financial support, AfriSam Aggregate for donating the ballast, and Aveng Infrasat for the supply of railway sleepers.

**REFERENCES**


Han, X 2012. The role of particle breakage on the permanent deformation of ballast. Master of Engineering, Research Thesis. Department of Civil, Mining and Environmental Engineering, University of Wollongong.


Mechanical properties, durability characteristics and shrinkage of plain cement and fly ash concretes subjected to accelerated carbonation curing

R A Assagaf, S K Adekunle, S Ahmad, M Maslehuddin, O S B Al-Amoudi, S I Ali

The paper presents an experimental study on assessment of the effect of accelerated carbonation curing (ACC) on the performance of two concrete mixtures having the same mixture proportions but different cementitious materials (plain-cement and fly-ash-blended-cement). Different sets of specimens were cast utilising both concrete mixtures and were then subjected to ACC for ten hours at a constant pressure of 414 kPa (60 psi). After exposing the specimens to ACC, they were tested for weight gain, carbonation depth, compressive and tensile strengths, modulus of elasticity, water penetration depth, rapid chloride permeability, shrinkage, SEM and XRD. ACC of the concrete specimens for ten hours resulted in a significant weight gain with less than 2 mm of carbonation depth. Both mixtures gained compressive strength above 20 MPa after ten hours of ACC. The strength increased further when ACC-treated specimens were exposed to air, with a significant increase up to seven days for plain-cement concrete and up to 28 days for fly-ash-blended-cement concrete. Compared to reference moist-cured concretes, the ACC-treated concretes were found to exhibit a slightly lower long-term strength (15% for plain-cement and 5% for fly-ash-concrete). However, the overall performance of the ACC-treated concrete mixtures was comparable with the respective moist-cured concrete mixtures.

INTRODUCTION
Carbonation is a process in which the CO₂ present in the atmosphere penetrates concrete and reacts with calcium hydroxide Ca(OH)_2 to form calcium carbonates CaCO₃ (Rostami et al. 2012). This chemical reaction is considered harmful to concrete durability characteristics if it is allowed to occur for a long period, since it is associated with a significant decrease in the pH of concrete, which in turn leads to the initiation of corrosion of steel bars embedded in the concrete when the concrete pH falls below a threshold value of about 10. However, accelerated carbonation curing (ACC), which involves exposing
Concrete to CO₂ under pressure for a short duration of time at an early age (after a few hours of casting), has been reported to improve the mechanical and durability properties of hardened concrete (Rostami et al. 2012). The CO₂ sequestered into young concrete reacts with Ca(OH)₂ and C-S-H generated by the cement hydration process, leading to the formation of geologically stable calcium carbonates (Rostami et al. 2012; Fernández Bertos et al. 2004; Mo & Panesar 2013; Shao et al. 2014). Unlike the long-term natural carbonation process of hardened concrete, which has been the focus of several research studies on concrete durability, ACC eliminates the loss-of-alkalinity problem, since the accompanying reduction in pH occurs to a negligible depth below the concrete surface, rather than in the concrete core. Therefore, reinforcing steel bars are safe from de-alkalisation-induced reinforcement corrosion (Rostami et al. 2012; Shao et al. 2014; Monkman & Shao 2010).

ACC changes the mineralogy, morphology and microstructure of concrete, leading to an increase in the density of concrete, which implies higher strength and durability, compared to that offered by the microstructure of concrete developed by conventional curing (Rostami et al. 2012; Mo & Panesar 2013; Pizzol et al. 2014). In addition, ACC involving sequestration of CO₂ in concrete helps in reducing the carbon footprint, mitigating the environmental problems associated with cement and concrete production.

Many factors affect the effectiveness of ACC, such as the concentration of CO₂, temperature, relative humidity, pressure in the carbonation chamber, exposure duration to CO₂, the age of concrete when exposed to CO₂, type and amount of binders, and water/cement ratio (Fernández Bertos et al. 2004; Chen et al. 2011; El-hassan et al. 2013; Mohammed et al. 2014; Shao & Monkman 2006; Zhan et al. 2013a; Zhan et al. 2013b). Kashef-Haghighi and Ghoshal (2013) found that CO₂ uptake could be increased significantly through the use of cements having higher amounts of reactive minerals and more fineness, which would provide a higher reactive surface area. Carbonation shrinkage resulting from ACC can be reduced by incorporating mineral admixtures such as slag (Monkman & Shao 2010). Rostami et al. (2012) reported that keeping specimens in the air for some time before exposure to CO₂ is very important to allow better diffusion of CO₂ into concrete.

The consumption of CO₂, captured from the manmade stationary sources of CO₂ emissions and stored for usage (carbon capture), in ACC of concrete would significantly help in reducing the global greenhouse gas emission and, therefore, the problems around global warming, which are of great environmental concern. About 1.5 million tons of CO₂ can be sequestered in concrete products made with 16.4 million tons of cement in the USA, based on about 9% of CO₂ by mass of cement (Kashef-Haghighi & Ghoshal 2013).

The present study was conducted to investigate the mechanical properties, durability characteristics, shrinkage and microstructure of plain-cement and flyash-blended-concrete mixtures cured using accelerated carbonation. The main objective of the present work was to explore the possibility of using ACC as an alternative curing method that can be adopted by precast concrete industries.

**EXPERIMENTAL PROGRAMME**

**Materials**

ASTM C 150 Type I Portland cement with a specific gravity of 3.15 was used in this study to prepare the concrete specimens. A class F fly ash was used to prepare the concrete mixture, consisting of a blend of Portland cement and fly ash. Table 1 shows the chemical compositions of Portland cement and fly ash used in this work.

Coarse aggregate with a maximum size of 12 mm, specific gravity of 2.60 and water absorption of 1.4% was used. Dune sand with a specific gravity of 2.56 and water absorption of 0.4% was used as fine aggregate. Figure 1 shows the grading curves of coarse and fine aggregates. CO₂ gas with 99.9% purity was used for carrying out ACC of the concrete specimens.

**Concrete mixtures**

Two concrete mixtures having the same mixture proportions, but with different cementitious materials, were considered in this study. The first mixture was prepared with a plain Portland cement as a cementitious material, and this concrete mixture was abbreviated as plain-cement concrete (PCC). In the second mixture, a blend of 20% fly ash and 80% Portland cement was used as cementitious material and this concrete mixture was abbreviated as flyash-blended-cement-concrete (FA-BCC). Table 2 shows the weights of the constituent materials for preparing one cubic metre

### Table 1 Chemical compositions of cement and fly ash (weight %)

<table>
<thead>
<tr>
<th>Component</th>
<th>Portland cement</th>
<th>Fly ash</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaO</td>
<td>64.35</td>
<td>8.38</td>
</tr>
<tr>
<td>SiO₂</td>
<td>22.00</td>
<td>45.3</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>5.64</td>
<td>34.4</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>3.80</td>
<td>2.37</td>
</tr>
<tr>
<td>K₂O</td>
<td>0.36</td>
<td>0.57</td>
</tr>
<tr>
<td>Na₂O</td>
<td>0.19</td>
<td>1.86</td>
</tr>
<tr>
<td>MgO</td>
<td>2.11</td>
<td>0.4</td>
</tr>
<tr>
<td>SO₃</td>
<td>–</td>
<td>0.46</td>
</tr>
<tr>
<td>LOI</td>
<td>–</td>
<td>3.5</td>
</tr>
</tbody>
</table>

**Table 2** shows the weights of the constituent materials for preparing one cubic metre of ACC in concrete.
moist-curing was carried out by sandwich-preliminary study (Ahmad et al. 2017). The crete mixture was determined by crushing initial compressive strength of each con-
crement in layers of burlap in a curing chamber, which was kept continu-
cre cement was moist-cured for seven days. ACC was carried out by exposing the specimens to CO₂ exposure duration, as reported in a previous experimental study reported elsewhere (Ahmad et al. 2017). The inner diameter and height of the purpose-built ACC cylindrical chamber were 400 and 500 mm, respectively. Three holes were made through the wall of the chamber. The inlet hole was connected to the CO₂ cylinder, while the second hole was connected to a pressure gauge to measure the pressure in the ACC chamber. The outlet hole was made to flush out the CO₂ from the chamber after curing. To ensure operational safety, the chamber was made of steel with a wall thickness of 8 mm.

Preparation and curing of concrete specimens
The specimens made of both concrete mixtures were demolded after 18 hours of casting, and the specimens of each of the two concrete mixtures were divided into two sets. One set of specimens was cured by ACC for ten hours, while the other set was moist-cured for seven days. ACC was carried out by exposing the specimens to CO₂ in the ACC chamber where a constant pressure of 414 kPa (60 psi) was maintained for a period of ten hours, in accordance with the optimum levels of pressure and CO₂ exposure duration, as reported in a preliminary study (Ahmad et al. 2017). The moist-curing was carried out by sandwiching the specimens in layers of burlap in a curing chamber, which was kept continu-
ously moist for a period of seven days. The initial compressive strength of each concrete mixture was determined by crushing four cubes of 50 mm size immediately after demolding.

Prior to applying ACC, all specimens were weighed using a precision electronic balance with a resolution of 0.01 g. Thereafter, the specimens were arranged inside the ACC chamber on a perforated platform. The flow control valve of the CO₂ gas cylinder was then released while the chamber outlet was still open. This step was continued for one minute to clean the chamber before ACC commenced. Subsequently, the outlet was closed, and the pressure regulator was adjusted to maintain a constant ACC pressure of 414 kPa (60 psi). The ACC process continued for ten hours before the cured specimens were taken out of the chamber. The post-ACC weight gain was recorded.

After the curing process for both ACC and moist-curing regimes, the specimens were tested to assess their mechanical properties, durability characteristics, shrinkage behaviour and microstructure.

Microstructure and identification of carbonation products
The morphology of the carbonated hydration products was studied by examining the secondary electron image (SEI) of near-surface zones of fractured concrete samples under a JEOL (JSM-6610LV model) scanning electron microscope (SEM), which was equipped with an energy dispersive spectroscopy (EDS) detector. Additionally, crystalline phases present in the ACC-cured specimens were analysed by X-ray diffraction (XRD), using a Rigaku (Ultima IV model) X-ray diffractometer with Cu-Kα radiation (λ = 1.5418 Å), over a 2θ range of 10 to 70° in steps of 0.02° 20. The near-surface samples used for the microstructure studies in this work were carefully collected to ensure the exclusion of coarse aggregates in order to isolate only the mortar phase of the concrete mixtures.

RESULTS AND DISCUSSION
Weight gain observations
For each of the two concrete mixtures, the weight gain was calculated after exposing 20 cube specimens to ACC for ten hours at 414 kPa. The specimens were weighed before and after ACC to determine the average CO₂ uptake of each mixture, expressed as percentage weight gain by mass of cement. Figure 2 shows the weight gain for the two concrete mixtures considered in this study. It can be seen from Figure 2 that FA-BCC had a 13% lower weight gain, compared to PCC. This can be attributed to the availability of a lower amount of hydration-generated Ca(OH)₂ for CO₂ uptake in FA-BCC, where 20% of Portland cement was replaced by fly ash.

Carbonation depth
After exposure to ACC, the carbonation depths were measured by splitting the specimens into two and then spraying phenolphthalein solution on the split surfaces of concrete. The carbonation depth was indicated by the thin outer layer of the discoloured portion of the concrete surface. Figure 3 schematically shows the carbonation profile, indicated by the portion with phenolphthalein staining (represented by the black colouring) and the colourless portion (represented by white colouring) on the fractured surface of a concrete.

![Figure 2 Weight gain in PCC and FA-BCC as percentage of cement mass](image-url)

<table>
<thead>
<tr>
<th>Material</th>
<th>Quantity (kg)</th>
<th>PCC</th>
<th>FA-BCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland cement</td>
<td></td>
<td>375</td>
<td>300</td>
</tr>
<tr>
<td>Fly ash</td>
<td>–</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Water (effective)</td>
<td>170</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>Water (gross)</td>
<td>194</td>
<td>194</td>
<td></td>
</tr>
<tr>
<td>Coarse aggregate</td>
<td>1 074</td>
<td>1 074</td>
<td></td>
</tr>
<tr>
<td>Dune sand</td>
<td>716</td>
<td>716</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Weights of the constituent materials for 1 m³ of concrete mixture

![Table 2](image-url)
specimen after exposed to ACC. For each sample, the carbonation depth was taken as the average value of the largest CO₂ penetration depth measured on all sides of the sample, as indicated in Figure 3.

Table 3 presents the average carbonation depths in mm for the two concrete mixtures considered in this study. It can be observed from Table 3 that the carbonation depth was around 2 mm for both concrete mixtures studied. However, FA-BCC experienced a slightly (11%) higher carbonation depth than PCC. A reverse of this observation might be expected, because a higher CO₂ uptake was recorded for the plain cement mixture. However, it is known that the pozzolanic action of fly ash is a slow process. Therefore, after 18 hours of casting when ACC commenced, the lower cement content in the FA-BCC would result in higher gas porosity because of the lower density of the primary C-S-H gel in the mixture. In addition, for the same reason (lower cement content), the lower amount of Ca(OH)₂ available for CO₂ uptake also poses less hindrance to further penetration at later stages of carbonation. Hence, the higher penetration depth in the fly ash mixture, despite its lower CO₂ uptake, can be justified. Subsequently, the lower reduction of CO₂ uptake of the fly ash mixture (13%), compared to the 20% reduction of cement content, can be explained by the deeper reach of the CO₂ front.

It is worth noting that the carbonation depth obtained using the method described earlier is an underestimate of the true depth of the carbonation front. Phenolphthalein indication only highlights the region with pH lower than 10. This implies that the measured average depth does not include the carbonated regions that are deeper than the phenolphthalein-highlighted zone of which the pH levels are higher than the sensitive range of phenolphthalein indicator. Nevertheless, the actual depth of carbonation is not expected to exceed two to three times that obtained through phenolphthalein indication measurements, which does not constitute a threat of corrosion to a steel bar embedded in a reinforced concrete element made with these mixtures, since the usual minimum cover to rebars is 20 mm. In addition, ACC of concrete elements results in densification of the thin carbonated surface layer (or skincrete). The densification of skincrete is expected to slow down the diffusion rate of CO₂ into concrete through the skincrete, leading to higher resistance to natural carbonation during the service life of the concrete element. Further, subsequent hydration of CO₂-cured concrete (upon contact with water) re-alkanises the concrete to pH in excess of 12.0 (Zhang & Shao 2016), offering additional protection of CO₂-cured concrete from natural carbonation during the service life.

**Evolution of compressive strength**

For concrete specimens subjected to ACC regime, the first test was conducted directly after ten hours of ACC, while the first test on specimens exposed to moist-curing regime was carried out after seven days of moist-curing. Subsequently, concrete specimens from both curing regimes were subjected to air-curing under laboratory conditions and were tested after 7, 14, 28 and 90 days of additional air-curing. The compressive strength development of PCC and fly ash FA-BCC for both ACC and moist-curing are shown in Figures 4 and 5. It can be observed from Figures 4 and 5 that both PCC and FA-BCC gained a compressive strength of more than 20 MPa only after subjecting them to ten hours of the ACC.

As can be seen from Figure 4, the ACC specimens achieved a higher compressive strength during the first seven days of air exposure. About 63% increase in the compressive strength due to the air-curing for the first seven days after ACC was recorded. However, very little benefit of post-ACC air-curing was observed after the first seven days of air exposure, as is evident from the nearly flat portion of strength evolution curve shown in Figure 4. A similar trend of strength evolution was observed for moist-cured specimens. An increase of about 30% in compressive strength was noted when moist-cured specimens were exposed to air for the first seven days. As in the case of ACC, the benefit of air exposure after moist-curing was found to be insignificant beyond seven days of air exposure.

It is important to note from Figure 4 that the strength of ACC specimens after seven days in the air was found to be about 14% higher than the strength of seven days moist-cured specimens. However, the strength of moist-cured specimens kept in the air for seven days was found to be around 10% higher than the strength of ACC specimens kept in the air for 14 days. This can be explained by the fact that the carbonation process is of exothermic nature. The heat generated from the process tends to increase the cement hydration rate as ACC progresses. Additionally, during the post-ACC air exposure, the carbonated layer around the surface of the ACC specimen significantly reduced the evaporation of moisture from the core of the specimens, which resulted in continuation of the hydration process until most of the free concrete water had been consumed, resulting in a lower rate of strength gain. On the contrary, in the specimens subjected to seven days moist-curing, the rate of strength gain during the moist-curing period was slower

---

**Figure 3** Schematic representations of phenolphthalein-stained concrete surfaces

**Table 3** The depth of carbonation for ACC specimens

<table>
<thead>
<tr>
<th>Mixture ID</th>
<th>Average depth of carbonation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCC</td>
<td>1.8</td>
</tr>
<tr>
<td>FA-BCC</td>
<td>2.0</td>
</tr>
</tbody>
</table>
than that of ACC specimens exposed to the air for seven days, due to the absence of carbonation-induced heating. Due to the slow rate of hydration, the amount of residual moisture available in the pores of moist-cured specimens would be more than that of ACC specimens (because ACC specimens consumed almost entire pore water for further hydration during the first seven days of air exposure). This therefore explains the observed higher strength gain during air exposure of moist-cured specimens compared to ACC specimens.

It can further be seen that the difference in the strength of ACC and moist-cured specimens is almost constant (about 15%) during the long-term air exposure. Therefore, it may be concluded that the difference in the strength of ACC and moist-cured specimens can be taken as 15% during the service life of the concrete. It is worth mentioning that the ACC concrete achieved a strength of 27 MPa only after ten hours of accelerated carbonation, making it possible to handle the carbonation-cured concrete only after ten hours of curing.

However, unlike the PCC made with only cement as binder, the strength of ACC specimens made of FA-BCC after exposure to air for seven days is almost similar to that of the seven days moist-cured specimens of FA-BCC. This observation may be explained by the relatively lower degree of carbonation achieved for FA-BCC due to the availability of a lower amount of Ca(OH)$_2$ as a result of its lower cement content. This relatively lower amount of Ca(OH)$_2$ was further reduced in secondary hydration with silica from the admixed fly ash. Due to the lower degree of carbonation, the enhancement of strength owing to the increased surface hardness was relatively lower, leading to the development of lower strength.

Unlike the case of PCC, where the rate of strength gain became almost negligible after seven days of air exposure of both ACC as well as moist-cured specimens, the rate of strength gain for both curing regimes in the case of FA-BCC was considerable up to 28 days of air exposure, as can be seen from Figure 5. This can be attributed to the slow rate of hydration due to the addition of fly ash to FA-BCC, which involved secondary hydration.

Furthermore, it can be seen from Figure 5 that the difference in the strength of ACC and moist-cured specimens is almost constant (by around 5%) during the long-term air exposure. Therefore, it may be concluded that the difference in the strength of ACC and moist-cured specimens of FA-BCC will be taken as only 5% during the service life of concrete. Again, it must be noted that the FA-BCC subjected to ACC achieved a strength of 24 MPa only after ten hours of carbonation.

**Splitting tensile strength**

Figure 6 shows the values of splitting tensile strength of both concrete mixtures at an age of 14 days for both ACC and moist-curing regimes. As can be seen from Figure 6, ACC appears to improve the tensile strength for PCC by 21% in comparison with the moist-curing regime, while a reversed situation is noted in the case of FA-BCC. However, even in the case of FA-BCC, the ACC regime maintained a tensile strength falling within the range recommended for a structural concrete mixture.

**Elastic modulus**

Figure 7 illustrates the 14-day chord modulus values for the two concrete mixtures subjected to ACC and moist-curing. Specimens subjected to ACC exhibited
a slightly lower stiffness than that of the moist-cured specimens for both mixtures. However, for both mixtures, the reduction was less than 10% with respect to moist-cured specimens. This slight stiffness reduction is still in line with the little reduction in strength of ACC specimens at 14 days, in relation to moist-cured specimens. Therefore, it can be concluded that the elastic modulus of ACC specimens is comparable to that of the moist-cured specimens.

**Water and chloride permeability**  
Figure 8 presents the water penetration depths, performed according to the DIN 1048 standard (DIN 2004), for the two concrete mixtures, PCC and FA-BCC. In each mixture, the test was conducted for both ACC and moist-cured specimens at 14 days, in relation to moist-cured specimens. Therefore, it can be concluded that the elastic modulus of ACC specimens is comparable to that of the moist-cured specimens.

The specimens made of PCC and subjected to ACC had a slightly lower water penetration depth than that of the moist-cured specimens. By making a connection to the higher CO₂ uptake in PCC and lower CO₂ penetration depth (as discussed earlier), ACC specimens are expected to exhibit higher surface layer density. This is in agreement with the claim that the carbonation of concrete makes the microstructure of concrete surface denser, thereby decreasing the porosity of the surface layer of concrete that resists fluid penetration (Ahmad et al. 2017). Although the improvement of the surface density is not significant, as indicated by water penetration depth, it can be inferred that ACC does not have negative effects on the properties of PCC.

On the other hand, a higher water penetration depth was noted in specimens made of FA-BCC and subjected to ACC compared to moist-cured specimens. A converse of the case for PCC may be used to explain this observation. FA-BCC exhibited lower CO₂ uptake than PCC, as a result of a lower amount of Ca(OH)₂ at the time of ACC, and for the same reason it also exhibited a higher CO₂ penetration depth. The combined effects of these two factors tend to reduce the surface layer density of FA-BCC ACC specimens, hence the recorded larger water penetration depth.

Figure 9 presents the results of the rapid chloride permeability test, performed according to the ASTM C1202 standard (ASTM 2012), for the two concrete mixtures. Even though the results are in the same range of ‘moderate permeability’ according to the referenced standard’s classification, the behaviour pattern recorded here is similar to that of the water permeability test.
Drying shrinkage

Figures 10 and 11 show the variations of drying shrinkage strain with time for ACC and moist-cured specimens belonging to PCC and FA-BCC mixtures, respectively. For the two concrete mixtures, the recorded drying shrinkage in ACC specimens was higher than that in moist-cured specimens, as is evident from Figures 10 and 11. Higher shrinkage in ACC specimens may be attributed to the shrinkage action associated with the chemical reactions involved in carbonation of the surface concrete, as well as the accompanying moisture loss to the exothermic process. It should be stated that the ACC-induced shrinkage can be controlled by spraying water on carbonated concrete immediately after ACC to compensate for the water loss during the ACC process which may significantly reduce the shrinkage.

Concrete microstructure characterisation

Figure 12 shows a micrograph of a fractured specimen exposed to ACC. The lower edge of the image is the edge of the concrete sample exposed to ACC. The red line indicates the actual profile of the carbonated area penetrated by CO₂. The dense structure of hydrates shown in Figure 12 is attributed to the formation of CaCO₃ which was caused by the accelerated carbonation of Ca(OH)₂ (portlandite) and conventional calcium silicate hydrate (C-S-H). The intermixing between CaCO₃ and C-S-H may be explained by the fact that the primary C-S-H was highly porous after about 18 hours of casting when the ACC was commenced. Therefore, the pores were available for diffusion of CO₂ and its associated reaction with the portlandite formed during the primary hydration process, resulting in the formation of CaCO₃. Based on the scale of the SEM image, the carbonation depth was estimated as 80 µm at the section imaged. Obviously, ACC reduced the permeability significantly by filling the pores and minor cracks with CaCO₃. Furthermore, this layer may reduce the evaporation of the internal water in the long term, imparting a self-curing ability to the concrete. All these explain the improved properties of the concrete cured with the ACC method.

In addition to the SEM imaging, the chemistry of selected areas of carbonated regions of near-surface concrete samples was also examined through quantitative EDS analysis (QEDS). Figure 13(a) shows the EDS profile of the area marked ‘spectrum 8’ in Figure 12 for a PCC sample exposed to ACC.
The EDS shown in Figure 13(a) indicates a high carbon content of 18.9% and a low amount of silica at 3.1%. Since the mortar phase sample examined was free of limestone aggregates (as seen in Figure 12), the high carbon content captured by the QEDS implies that the examined region of the carbonated specimen contained a significant amount of carbonation products. Figure 14(a) depicts the mineralogical composition of concrete specimens cured with the ACC method. The presence of calcite of about 25% by mass of the tested sample was noted. This observation implies that most of the high amount of carbon indicated by QEDS was from calcite that resulted from the reaction of CO₂ with portlandite, as well as carbonated C-S-H gels.

A micrograph of a fractured sample of FA-BCC concrete specimen exposed to ACC is shown in Figure 15. Some carbonation products can be recognised, but the amount of CaCO₃ produced was less than that of products can be recognised, but the amount is shown in Figure 15. Some carbonation and deeper penetration of CO₂, as well as carbonated C-S-H gels. Figure 13(b) shows the EDS of the area shown in Figure 15 (Spectrum 4). The EDS established the fact that ACC of FA-BCC resulted in a lower degree of carbonation than that in ACC, since the amount of silica detected was slightly higher (8.7%) compared to the case of ACC, which resulted from the reaction of CO₂ with portlandite, as well as carbonated C-S-H gels.

Therefore, it can be concluded that the negative effect of ACC on the measured durability indices (water permeability and chloride permeability) of the FA-BCC, as a combined result of a lower degree of carbonation and deeper penetration of CO₂, has been well captured by morphological and chemical analyses.

**CONCLUSIONS**

From the results presented and the explanations offered for the reported observations, the following conclusions can be drawn:

- The carbonation depth was in the order of a few mm for both concrete mixtures, which may be considered a safe depth for a reinforced concrete element against reinforcement corrosion.

- Carboneation depth and weight gain analyses have established that the higher strength gain in plain-cement mixture can be attributed to a higher degree of carbonation coupled with lower depth of CO₂ penetration.

- An increase in the compressive strength by around 60% was recorded in both mixtures when they were exposed to air-curing for seven days after ten hours of ACC.

- ACC specimens exhibited 5 to 15% lower long-term strength compared to moist-cured specimens, indicating that the ultimate strength of concrete exposed to ACC is not much different from that of the concrete exposed to seven days of moist-curing.

- Although the shrinkage of ACC-treated specimens was generally higher than that of moist-cured specimens for the two concrete mixtures studied, post-ACC water spraying was required to compensate for the water loss during the ACC process.

- The tensile strength and modulus of elasticity of the ACC specimens were found to be comparable to those of moist-cured specimens. However, the measured durability indices indicated some negative effect of ACC on the durability characteristics of fly-ash-concrete, while the plain-cement-concrete exhibited improved durability characteristics.

- The negative effect of ACC on the measured durability indices of the fly-ash-concrete was well captured by morphological and chemical analyses, through SEM and XRD.

**ACKNOWLEDGEMENTS**

The authors gratefully acknowledge the financial support provided by King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, under research grant Projects No RG1323-1 and RG1323-2. The logistical support of the Department of Civil and Environmental Engineering, and the Research Institute at the same university is also acknowledged with appreciation.

**REFERENCES**

Ahmad, S, Assaggaf, R A, Maslehuddin, M, Al-amoudi, O S B, Adekunle, S K & Ali, S I 2017. Effects of carbonation pressure and duration on strength...
Figure 14 XRD of specimens exposed to ACC

Figure 15 SEM micrograph of FA-BCC specimens exposed to ACC


